Arithmetic Average Strike Option

Vector Risk Pty Ltd

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1 Formula

Henderson and Wojakowski (2002) discuss a symmetry relationship between fixed and floating strike averaging options, allowing *forward starting* arithmetic average strike options to be valued as manipulated forward starting arithmetic average rate options.

At expiry, the holder of a call (or put) option has the right to buy (or sell) λ units of the underlying, and sell (or buy) the accumulated average.

The value of a forward starting arithmetic average strike option is thus given by

$$\mathrm{ASO}(\phi, \mathbf{A}_\tau, \lambda) = -\phi e^{-q_T T} \sum_{i=1}^{n} w_i \mathbb{E}\left(S_{\tau_i}\right) N\left(\phi \frac{Y_X^* - \mathrm{Cov}\left(\ln S_{\tau_i}, Y\right) - \mathbb{E}(Y)}{\sqrt{\mathrm{Var}(Y)}}\right) + \phi\lambda S e^{-q_T T} N\left(\phi \frac{Y^* - \mathbb{E}(Y)}{\sqrt{\mathrm{Var}(Y)}}\right),$$

where $\mathbf{A}_\tau = \left\{\left(w_1, S_{\tau_1}^*\right), \ldots, \left(w_n, S_{\tau_n}^*\right)\right\}$ is the set of scaled weights and manipulated underlying values at each averaging date t_i, $1 \le i \le n$ with

$$S_{\tau_i}^* = S \exp\left\{\left(q_{\tau_i} - r_{\tau_i} - \frac{\sigma_{\tau_i}^2}{2}\right) + \sigma_{\tau_i} \widetilde{\mathcal{W}}_{\tau_i}^*\right\}.$$

Y^* is found by solving[1]

$$\sum_{i=1}^{n} w_i \mathbb{E}\left(S_{\tau_i}^*\right) \exp\left\{\frac{\mathrm{Cov}\left(\ln S_{\tau_i}, Y\right)}{\mathrm{Var}(Y)}\left[Y_X^* - \mathbb{E}(Y)\right] - \frac{1}{2}\frac{\left[\mathrm{Cov}\left(\ln S_{\tau_i}^*, Y\right)\right]^2}{\mathrm{Var}(Y)}\right\} = \lambda S,$$

and where T is time to maturity

$$\tau_i = T - t_i \qquad r_{\tau_i} = \frac{r_T T - r_{t_i} t_i}{T - t_i} \qquad q_{\tau_i} = \frac{q_T T - q_{t_i} t_i}{T - t_i} \qquad \sigma_{\tau_i}^2 = \frac{\sigma_T^2 T - \sigma_{t_i}^2 t_i}{T - t_i}$$

$$\mathbb{E}\left(S_{\tau_i}^*\right) = S e^{\left(q_{\tau_i} - r_{\tau_i}\right)\tau_i} \qquad \mathbb{E}(Y) = \sum_{i=1}^{n} w_i \left[\ln \mathbb{E}\left(S_{\tau_i}^*\right) - \frac{1}{2}\sigma_{\tau_i}^2 \tau_i\right]$$

$$\mathrm{Var}(Y) = \sum_{i=1}^{n} w_i \sigma_{\tau_i}^2 \tau_i \left(w_i + 2\sum_{j=1}^{n} w_j - 2\sum_{j=i}^{n} w_j\right) \qquad \mathrm{Cov}\left(\ln S_{\tau_n}^*, Y\right) = \sigma_{\tau_n}^2 \tau_n$$

$$\mathrm{Cov}\left(\ln S_{\tau_i}^*, Y\right) = \mathrm{Cov}\left(\ln S_{\tau_{i+1}}^*, Y\right) + \left(\sigma_{\tau_i}^2 \tau_i - \sigma_{\tau_{i+1}}^2 \tau_{i+1}\right)\left(\sum_{j=1}^{n} w_j - \sum_{j=i+1}^{n} w_j\right) \text{ (for } i < n\text{)}.$$

ϕ	Option Type
-1	Put
1	Call

[1] Any root finding method is acceptable

2 Properties of Instrument

The payoff for an arithmetic average strike call option at expiry, when the holder has the right to buy λ units of the underlying, is given by

$$C_T = (\lambda S_T - A_T)^+ \,, \tag{1}$$

where the weighted arithmetic mean of the underlying at n pre-defined observation times t_i is given by

$$A_T = \sum_{i=1}^{n} w_i S_{t_i}, \tag{2}$$

where $(w_i,\, S_{t_i})$ is the (weight, underlying) pair at t_i.

Similarly, the payoff for an arithmetic average strike put option at expiry, where the holder has the right to sell λ units of the underlying, is given by

$$P_T = (A_T - \lambda S_T)^+ \,. \tag{3}$$

Bibliography

Vicky Henderson and Rafal Wojakowski. On the equivalence of floating and fixed-strike asian options. *Journal of Applied Probability*, 39(2):391–394, 2002.