Black-Scholes Generalised

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April 06, 2017

Version 8.0.7905

1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Volatility till T	σ	0^+	$+\infty$	
Time to maturity	T	0^+	T_s	
Continuous risk-free interest rate till T_s	r_s	0^+	$+\infty$	
Continuous secondary rate till T_s	q_s	0^+	$+\infty$	
Time to settlement	T_s	T	$+\infty$	
Put or Call	*indicator*	–	–	"P", "C"

Table 1: Inputs for Black Scholes Generalised pricing function

2 Formula

Black and Scholes (1973) published the famous Black-Scholes formula, which prices a plain vanilla option. Merton (1973) extended this to allow for stock with continuous dividend, giving a generalised formula taking a generic continuous 'secondary-rate'.

Our Risk Engine uses the following Black-Scholes Generalised pricing formula, which includes the settlement period,

$$\boxed{f = \phi S e^{-q_s T_s} N(\phi d_1) - \phi X e^{-r_s T_s} N(\phi d_2),} \tag{1}$$

where

$$d_1 = \frac{\ln \frac{S}{X} + (r_s - q_s) T_s + \frac{\sigma^2}{2} T}{\sigma \sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}. \tag{2}$$

ϕ	Option Type
-1	Put
1	Call

3 Properties of Instrument

The payoff of a plain vanilla option is determined by the relative positions of the option strike price X, and the value of the underlying S at maturity T with S_T representing the value of the underlying at maturity. If we denote the value of the underlying at settlement T_s as S_{T_s}, the holder of the option will only exercise the option if at maturity, the forward contract that settles at the option settlement with strike price X has positive value. For a call option, the forward contract has positive value if the forward price of the underlying is greater than the strike price, i.e. $\mathbb{E}\left(S_{T_s} \mid S_T\right) > X$ and for a put option, the forward contract has positive value if the forward price of the underlying is less than the strike price, i.e. $\mathbb{E}\left(S_{T_s} \mid S_T\right) < X$, where forward price of the underlying at settlement is

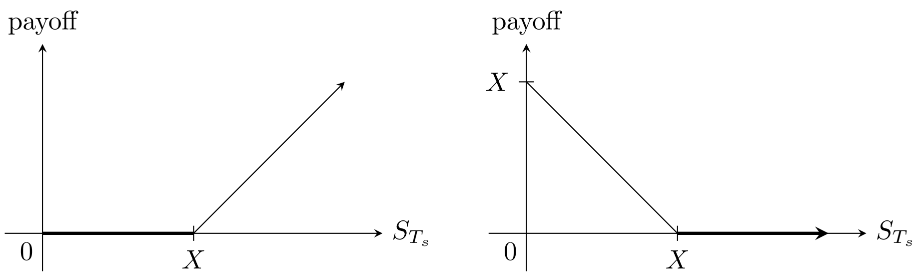
$$\mathbb{E}\left(S_{T_s} \mid S_T\right) = S_T e^{(r_\tau - q_\tau)\tau},$$

where $\tau = T_s - T$, is the time between maturity and settlement, with r_τ and q_τ being the forward continuous risk-free rate and forward continuous second rate for this period.

The payff at settlement for a vanilla option is illustrated in Table 2 and Figure 1.

Option Type	Condition	Payoff
Call	$S_T e^{(r_\tau - q_\tau)\tau} > X$	$S_{T_s} - X$
	$S_T e^{(r_\tau - q_\tau)\tau} \leq X$	0
Put	$S_T e^{(r_\tau - q_\tau)\tau} \geq X$	0
	$S_T e^{(r_\tau - q_\tau)\tau} < X$	$X - S_{T_s}$

Table 2: Payoff at settlement for vanilla option



(a) Call

(b) Put

Figure 1: Payoff at settlement for vanilla option

Bibliography

Fischer Black and Myron Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81 (3):637–654, 1973.

Robert Cox Merton. Theory of rational option pricing. *Bell Journal of Economics and Management Science*, 4(1): 141–183, Spring 1973.

Glossary

Risk Engine The Vector Risk market risk and credit risk system.