

# Binary Asset with Double Barrier Option

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April 06, 2017

Version 8.0.7905

## 1 Input to Function

<i>Description</i>	<i>Symbol</i>	<i>min</i>	<i>max</i>	<i>Reasonable range</i>
Underlying	$S$	$0^+$	$+\infty$	
Strike	$X$	$0^+$	$+\infty$	
Lower barrier level	$L$	$0^+$	$< U$	
Upper barrier level	$U$	$> L$	$+\infty$	
Continuous risk-free interest rate	$r$	$0^+$	$+\infty$	
Continuous secondary rate	$q$	$0^+$	$+\infty$	
Volatility	$\sigma$	$0^+$	$+\infty$	
Time to maturity	$T$	$0^+$	$+\infty$	
Put or Call	<i>indicator</i>	–	–	“P”, “C”
In or Out		–	–	“I”, “O”

Table 1: Inputs for Binary Asset with Double Barrier Option pricing function

## 2 Formula

The value of knock-out type binary asset with double barrier option is

$$Se^{-qT} \sum_{n=-\infty}^{\infty} \left\{ \left( \frac{U^n}{L^n} \right)^{2(\mu+1)} [N(a_1) - N(a_3)] - \left( \frac{L^{n+1}}{SU^n} \right)^{2(\mu+1)} [N(a_5) - N(a_7)] \right\},$$

where

$$a_1 = \frac{\ln \frac{SU^{2n}}{\alpha L^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}}, \quad a_3 = \frac{\ln \frac{SU^{2n}}{\beta L^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}}$$

$$a_5 = \frac{\ln \frac{L^{2n+2}}{\alpha SU^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}}, \quad a_7 = \frac{\ln \frac{L^{2n+2}}{\beta SU^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma \sqrt{T}}$$

$$\mu = \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2},$$

Option Type	$\phi$	$\alpha$	$\beta$
Put	-1	$L$	$\min(X, U)$
Call	1	$\max(X, L)$	$U$

unless the option will never be in the money, i.e.  $X \geq U$  for call option or  $X \leq L$  for put option, which has value of zero.

The value of a knock-in type binary asset with double barrier option can be found by in-out parity.

### 3 Properties of Instrument

A binary asset with double barrier option is knocked-in or knocked-out if the underlying price touched the lower boundary  $L$  or the upper boundary  $U$  prior to expiration.

For a knock-out type option, the payoff is as for a binary asset-or-nothing option, provided that both barriers were *not* touched during the life of the option, and zero otherwise.

For a knock-in type option, the payoff is as for a binary asset-or-nothing option, provided that at least one of the barriers *was* touched during the life of the option, and zero otherwise.