Binary Cash with Double Barrier Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Cash amount payoff	K	0^+	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	
Put or Call		–	–	"P", "C"
Up or Down	*indicator*	–	–	"U", "D"
In or Out		–	–	"I", "O"

Table 1: Inputs for Binary Cash with Double Barrier Option pricing function

2 Formula

The value of knock-out type binary cash with double barrier option is

$$Ke^{-rT}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{U^n}{L^n}\right)^{2\mu}\left[N\left(a_2\right)-N\left(a_4\right)\right]-\left(\frac{L^{n+1}}{SU^n}\right)^{2\mu}\left[N\left(a_6\right)-N\left(a_8\right)\right]\right\},$$

where

$$a_2=\frac{\ln\frac{SU^{2n}}{\alpha L^{2n}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad a_4=\frac{\ln\frac{SU^{2n}}{\beta L^{2n}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$a_6=\frac{\ln\frac{L^{2n+2}}{\alpha SU^{2n}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad a_8=\frac{\ln\frac{L^{2n+2}}{\beta SU^{2n}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$\mu=\frac{r-q-\frac{\sigma^2}{2}}{\sigma^2},$$

Option Type	ϕ	α	β
Put	-1	L	$\min(X,U)$
Call	1	$\max(X,L)$	U

unless the option will never be in the money, i.e. $X \geq U$ for call option or $X \leq L$ for put option, which has value of zero.

The value of a knock-in type binary cash with double barrier option can be found by in-out parity.

3 Properties of Instrument

A binary cash with double barrier option is knocked-in or knocked-out if the underlying price touched the lower boundary L or the upper boundary U prior to expiration.

For a knock-out type option, the payoff is as for a binary cash-or-nothing option, provided that both barries were *not* touched during the life of the option, and zero otherwise.

For a knock-in type option, the payoff is as for a binary cash-or-nothing option, provided that at least one of the barriers *was* touched during the life of the option, and zero otherwise.