Binary Cash with Single Barrier Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Barrier level	H	0^+	$+\infty$	
Cash amount payoff	K	0^+	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	
Put or Call		–	–	"P", "C"
Up or Down	*indicator*	–	–	"U", "D"
In or Out		–	–	"I", "O"

Table 1: Inputs for Binary Cash with Single Barrier Option pricing function

2 Formula

For a *binary cash with single barrier* option, the value is given by[1]

1) Down-and-in call ($S > H$)
   Payoff: K if $S_t \leq H$ for some $0 \leq t \leq T$ and $S_T > X$, zero otherwise.
   Value: ($X > H$): B_3 $\quad \eta = 1$
   Value: ($X < H$): $B_1 - B_2 + B_4$ $\quad \eta = 1, \quad \phi = 1$

2) Up-and-in call ($S < H$)
   Payoff: K if $S_t \geq H$ for some $0 \leq t \leq T$ and $S_T > X$, zero otherwise.
   Value: ($X > H$): B_1 $\quad \phi = 1$
   Value: ($X < H$): $B_2 - B_3 + B_4$ $\quad \eta = -1, \quad \phi = 1$

3) Down-and-in put ($S > H$)
   Payoff: K if $S_t \leq H$ for some $0 \leq t \leq T$ and $S_T < X$, zero otherwise.
   Value: ($X > H$): $B_2 - B_3 + B_4$ $\quad \eta = 1, \quad \phi = -1$
   Value: ($X < H$): B_1 $\quad \phi = -1$

4) Up-and-in put ($S < H$)
   Payoff: K if $S_t \geq H$ for some $0 \leq t \leq T$ and $S_T < X$, zero otherwise.
   Value: ($X > H$): $B_1 - B_2 + B_4$ $\quad \eta = -1, \quad \phi = -1$
   Value: ($X < H$): B_3 $\quad \eta = -1$

[1] Haug (2007) p.176 4.19.5 *Binary Barrier Options*

5) Down-and-out call $(S > H)$
Payoff: K if $S_t > H$ for all $0 \leq t \leq T$ and $S_T > X$, zero otherwise.

Value: $(X > H)$:	$B_1 - B_3$	$\eta = 1,$	$\phi = 1$
Value: $(X < H)$:	$B_2 - B_4$	$\eta = 1,$	$\phi = 1$

6) Up-and-out call $(S < H)$
Payoff: K if $S_t < H$ for all $0 \leq t \leq T$ and $S_T > X$, zero otherwise.

Value: $(X > H)$:	0		
Value: $(X < H)$:	$B_1 - B_2 + B_3 - B_4$	$\eta = -1,$	$\phi = 1$

7) Down-and-out put $(S > H)$
Payoff: K if $S_t > H$ for all $0 \leq t \leq T$ and $S_T < X$, zero otherwise.

Value: $(X > H)$:	$B_1 - B_2 + B_3 - B_4$	$\eta = 1,$	$\phi = -1$
Value: $(X < H)$:	0		

8) Up-and-out put $(S < H)$
Payoff: K if $S_t < H$ for all $0 \leq t \leq T$ and $S_T < X$, zero otherwise.

Value: $(X > H)$:	$B_2 - B_4$	$\eta = -1,$	$\phi = -1$
Value: $(X < H)$:	$B_1 - B_3$	$\eta = -1,$	$\phi = -1$

where

$$B_1 = Ke^{-rT} N(\phi d_2) \qquad B_2 = Ke^{-rT} N(\phi h_2)$$

$$B_3 = Ke^{-rT} \left(\frac{H}{S}\right)^{2\mu} N(\eta y_2) \qquad B_4 = Ke^{-rT} \left(\frac{H}{S}\right)^{2\mu} N(\eta y_4)$$

$$d_2 = \frac{\ln \frac{S}{X}}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T} \qquad h_2 = \frac{\ln \frac{S}{H}}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T}$$

$$y_2 = \frac{\ln \frac{H^2}{SX}}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T} \qquad y_4 = \frac{\ln \frac{H}{S}}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T}$$

$$\mu = \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2}.$$

3 Properties of Instrument

Reiner and Rubinstein (1991) introduced a set of formulae that can value binary cash with single barrier options. Binary cash with single barrier options are standard binary cash-or-nothing options, with a single barrier, where the payoff of the option is dependent on whether the barrier is touched. It can also be viewed as a standard single barrier option with binary cash payoff.

For a knock-out type option, the payoff is as for a binary cash-or-nothing option, provided the barrier was *not* touched during the life of the option, and zero otherwise.

For a knock-in type option, the payoff is as for a binary cash-or-nothing option, provided the barrier *was* touched during the life of the option, and zero otherwise.

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas.* McGraw Hill, New York, 2nd edition, 2007.

Eric Reiner and Mark Rubinstein. Unscrambling the binary code. *Risk*, 4(9):75–83, October 1991.