Complex Chooser Option

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1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Continuous risk-free interest rate until t	r	0^+	$+\infty$	
Continuous secondary rate until t	q	0^+	$+\infty$	
Volatility until t	σ	0^+	$+\infty$	
Time to decision date	t	0^+	$< \min(T_c, T_p)$	
Call strike	X_c	0^+	$+\infty$	
Continuous risk-free interest rate until T_c	r_c	0^+	$+\infty$	
Continuous secondary rate until T_c	q_c	0^+	$+\infty$	
Volatility until T_c	σ_c	0^+	$+\infty$	
Time to call maturity	T_c	$> t$	$+\infty$	
Put strike	X_p	0^+	$+\infty$	
Continuous risk-free interest rate until T_p	r_p	0^+	$+\infty$	
Continuous secondary rate until T_p	q_p	0^+	$+\infty$	
Volatility until T_p	σ_p	0^+	$+\infty$	
Time to put maturity	T_p	$> t$	$+\infty$	

Table 1: Inputs for Complex Chooser Option pricing function

2 Formula

The value of a complex chooser option is given by

$$\boxed{Se^{-q_c T_c} N_2(d_1, y_1; \rho_1) - X_c e^{-r_c T_c} N_2(d_2, y_2; \rho_1) - Se^{-q_p T_p} N_2(-d_1, -y_3; \rho_2) + X_p e^{-r_p T_p} N_2(-d_2, -y_4; \rho_2),}$$

where

$$d_1 = \frac{\ln \frac{S}{I} + \left(r - q + \frac{\sigma^2}{2}\right) t}{\sigma \sqrt{t}} \qquad d_2 = d_1 - \sigma \sqrt{t}$$

$$y_1 = \frac{\ln \frac{S}{X_c} + \left(r_c - q_c + \frac{\sigma_c^2}{2}\right) T_c}{\sigma_c \sqrt{T_c}} \qquad y_2 = y_1 - \sigma_c \sqrt{T_c}$$

$$y_3 = \frac{\ln \frac{S}{X_p} + \left(r_p - q_p + \frac{\sigma_p^2}{2}\right) T_p}{\sigma_p \sqrt{T_p}} \qquad y_4 = y_3 - \sigma_p \sqrt{T_p}$$

$$\rho_1 = \frac{\sigma \sqrt{t}}{\sigma_c \sqrt{T_c}} \qquad \rho_2 = \frac{\sigma \sqrt{t}}{\sigma_p \sqrt{T_p}},$$

and I satisfies the following equality

$$Ie^{-q_{1c}(T_c-t)}N(z_1) - Xe^{-r_{1c}(T_c-t)}N(z_2) = Xe^{-r_{1p}(T_p-t)}N(-z_3) - Ie^{-q_{1p}(T_p-t)}N(-z_4),$$

where

$$z_1 = \frac{\ln\frac{I}{X_c} + \left(r_{1c} - q_{1c} + \frac{\sigma_{1c}^2}{2}\right)(T_c - t)}{\sigma_{1c}\sqrt{T_c - t}} \qquad z_2 = z_1 - \sigma_{1c}\sqrt{T_c - t}$$

$$z_3 = \frac{\ln\frac{I}{X_p} + \left(r_{1p} - q_{1p} + \frac{\sigma_{1p}^2}{2}\right)(T_p - t)}{\sigma_{1p}\sqrt{T_p - t}} \qquad z_4 = z_3 - \sigma_{1p}\sqrt{T_p - t}$$

$$r_{1c} = \frac{r_c T_c - rt}{T_c - t} \qquad r_{1p} = \frac{r_p T_p - rt}{T_p - t}$$

$$q_{1c} = \frac{q_c T_c - qt}{T_c - t} \qquad q_{1p} = \frac{q_p T_p - qt}{T_p - t}$$

$$\sigma_{1c} = \sqrt{\frac{\sigma_c^2 T_c - \sigma^2 t}{T_c - t}} \qquad \sigma_{1p} = \sqrt{\frac{\sigma_p^2 T_p - \sigma^2 t}{T_p - t}}.$$

3 Properties of Instrument

Rubinstein (1991) introduced chooser options, allowing the holder to choose whether the option ‘becomes’ a standard call or put after a given time t. A complex chooser option refines this idea further, setting the call option to have strike X_c and time to maturity T_c, and the put option to have strike X_p and time to maturity T_p.[1]

The payoff of a complex chooser option at time t, ($t < T_c$, $t < T_p$) is thus

$$\max\left\{\mathrm{C}(S_t, X_c, T_c - t), \mathrm{P}(S_t, X_p, T_p - t)\right\},$$

where C() and P() are the values of a Generalised Black-Scholes call and put option respectively, with uncertain underlying S_t (the value of the underlying at time t), strikes X_c and X_p, and times to maturity $(T_c - t)$ and $(T_p - t)$.

We extend result of Rubinstein (1991) to obtain valuation function for the non-flat case.

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas*. McGraw Hill, New York, 2nd edition, 2007.

Mark Rubinstein. Options for the undecided. *Risk*, 4(4):43, April 1991.

[1] Haug (2007), p.129, 4.12.2 *Complex Chooser Options*