Compound Option

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1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike of compound option	x_1	0^+	$+\infty$	
Strike of underlying option	X_2	0^+	$+\infty$	
Continuous risk-free interest rate up to t_1	r_1	0^+	$+\infty$	
Continuous secondary rate up to t_1	q_1	0^+	$+\infty$	
Volatility up to t_1	σ_1	0^+	$+\infty$	
Time to maturity of compound option	t_1	0^+	$< T_2$	
Continuous risk-free interest rate up to T_2	r_2	0^+	$+\infty$	
Continuous secondary rate up to T_2	q_2	0^+	$+\infty$	
Volatility up to T_2	σ_2	0^+	$+\infty$	
Time to maturity of underlying option	T_2	$> t_1$	$+\infty$	
Compound Put or Call	*indicatorC*	–	–	"P", "C"
Underlying Put or Call	*indicatorU*	–	–	"P", "C"

Table 1: Inputs for Compound Option pricing function

2 Formula

The value of a compound option is given by

$$\phi\eta S e^{-q_2 T_2} N_2\left(\phi\eta a_1, \eta b_1; \phi\rho\right) - \phi\eta X_2 e^{-r_2 T_2} N_2\left(\phi\eta a_2, \eta b_2; \phi\rho\right) - \phi x_1 e^{-r_1 t_1} N\left(\phi\eta a_2\right),$$

where

$$a_1 = \frac{\ln\frac{S}{f} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1\sqrt{t_1}} \qquad a_2 = a_1 - \sigma_1\sqrt{t_1}$$

$$b_1 = \frac{\ln\frac{S}{X_2} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2\sqrt{T_2}} \qquad b_2 = b_1 - \sigma_2\sqrt{T_2}$$

$$\rho = \frac{\sigma_1\sqrt{t_1}}{\sigma_2\sqrt{T_2}},$$

ϕ	Compound Option	η	Underlying Option
-1	Put	-1	Put
1	Call	1	Call

and f solves

$$\eta f e^{-q_{12}(T_2-t_1)} N(\eta z_1) - \eta X_2 e^{-r_{12}(T_2-t_1)} N(\eta z_2) - x_1 = 0,$$

where

$$z_1 = \frac{\ln \frac{f}{X_2} + \left(r_{12} - q_{12} + \frac{\sigma_{12}^2}{2}\right)(T_2 - t_1)}{\sigma_{12}\sqrt{T_2 - t_1}} \qquad z_2 = z_1 - \sigma_{12}\sqrt{T_2 - t_1},$$

and r_{12}, q_{12} and σ_{12} denote the forward interest, secondary and volatility rates between time t_1 and T_2.

3 Properties of Instrument

Both Geske (1979) and Rubinstein (1991) published formulae for valuing compound options. Compound options are options-on-options with a strike price x_1 at time t_1 to either buy (call) or sell (put) another option (either call or put) with strike X_2 and maturity T_2. This can be broken down into four scenarios:

1) call-on-call: at t_1 the holder has the right to *buy* a T_2 maturing *call* option with strike X_2, for x_1,

2) call-on-put: at t_1 the holder has the right to *buy* a T_2 maturing *put* option with strike X_2, for x_1,

3) put-on-call: at t_1 the holder has the right to *sell* a T_2 maturing *call* option with strike X_2, for x_1, and

4) put-on-put: at t_1 the holder has the right to *sell* a T_2 maturing *put* option with strike X_2, for x_1.

We extend their work to obtain valuation function for the non-flat case.

Bibliography

Robert Geske. The valuation of compound options. *Journal of Financial Economics*, 7:63–81, 1979.

Mark Rubinstein. Double trouble. *Risk*, 5:73, December 1991.