Double Barrier Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	
Put or Call	*indicator*	–	–	"P", "C"
In or Out		–	–	"I", "O"

Table 1: Inputs for Double Barrier Option pricing function

2 Formula

The value of knock-out type double barrier option is

$$\boxed{\begin{aligned}\phi S e^{-qT} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{2(\mu+1)} \left[N(a_1) - N(a_3)\right] - \left(\frac{L^{n+1}}{SU^n}\right)^{2(\mu+1)} \left[N(a_5) - N(a_7)\right] \right\} \\ - \phi X e^{-rT} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{2\mu} \left[N(a_2) - N(a_4)\right] - \left(\frac{L^{n+1}}{SU^n}\right)^{2\mu} \left[N(a_6) - N(a_8)\right] \right\},\end{aligned}}$$

where

$$a_1 = \frac{\ln \frac{SU^{2n}}{\alpha L^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \qquad a_2 = a_1 - \sigma\sqrt{T}$$

$$a_3 = \frac{\ln \frac{SU^{2n}}{\beta L^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \qquad a_4 = a_3 - \sigma\sqrt{T}$$

$$a_5 = \frac{\ln \frac{L^{2n+2}}{\alpha S U^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \qquad a_6 = a_5 - \sigma\sqrt{T}$$

$$a_7 = \frac{\ln \frac{L^{2n+2}}{\beta S U^{2n}} + \left(r - q + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \qquad a_8 = a_7 - \sigma\sqrt{T}$$

$$\mu = \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2},$$

Option Type	ϕ	α	β
Put	-1	L	$\min(X, U)$
Call	1	$\max(X, L)$	U

unless the option will never be in the money, i.e. $X \geq U$ for call option or $X \leq L$ for put option, which has value of zero.

The value of a knock-in type double barrier option can be found by in-out parity.

3 Properties of Instrument

A double barrier option is knocked-in or knocked-out if the underlying price touched the lower boundary L or the upper boundary U prior to expiration.

For a knock-out type option, the payoff is as for a vanilla option, provided that both barries were *not* touched during the life of the option, and zero otherwise.

For a knock-in type option, the payoff is as for a vanilla option, provided that at least one of the barriers *was* touched during the life of the option, and zero otherwise.

Haug (1998)[1] states that double barrier options can be priced using the Ikeda and Kunitomo (1992) formula. However, the Ikeda and Kunitomo (1992) formula only works for cases where the strike price is between the lower and upper barriers. In our Risk Engine, a modified version of the Ikeda and Kunitomo (1992) formula derived from first principle is used.

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas*. McGraw Hill, New York, 1st edition, 1998.

Masayuki Ikeda and Naoto Kunitomo. Pricing options with curved boundaries. *Mathematical Finance*, 2(4):275–298, October 1992.

Glossary

Risk Engine The Vector Risk market risk and credit risk system.

[1] Haug (1998) p.72 2.10.2 *Double Barrier Options*