Double Barrier Asset-at-Expiry Option

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April 06, 2017

Version 8.0.7905

1 Input to Function

Description	Symbol	min	max	Reasonable range
Underlying	S	0^{+}	$+\infty$	
Lower barrier level	L	0^{+}	< U	
Upper barrier level	U	> L	$+\infty$	
Continuous risk-free interest rate	r	0^{+}	$+\infty$	
Continuous secondary rate	q	0^{+}	$+\infty$	
Volatility	σ	0^{+}	$+\infty$	
Time to maturity	T	0^{+}	$+\infty$	
In or Out	indicator	—	_	"I", "O"

Table 1: Inputs for Double Barrier Asset-at-Expiry Option pricing function

2 Formula

The value of a knock-out type double barrier asset-at-expiry option is given by

$$\left(Se^{-qT}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{U^{n}}{L^{n}}\right)^{2(\mu+1)}\left[N\left(a_{1}\right)-N\left(a_{3}\right)\right]-\left(\frac{L^{n+1}}{SU^{n}}\right)^{2(\mu+1)}\left[N\left(a_{5}\right)-N\left(a_{7}\right)\right]\right\}\right)$$

where

$$a_{1} = \frac{\ln \frac{SU^{2n}}{L^{2n+1}} + \left(r - q + \frac{\sigma^{2}}{2}\right)T}{\sigma\sqrt{T}} \qquad a_{3} = \frac{\ln \frac{SU^{2n-1}}{L^{2n}} + \left(r - q + \frac{\sigma^{2}}{2}\right)T}{\sigma\sqrt{T}} \\ a_{5} = \frac{\ln \frac{L^{2n+1}}{SU^{2n}} + \left(r - q + \frac{\sigma^{2}}{2}\right)T}{\sigma\sqrt{T}} \qquad a_{7} = \frac{\ln \frac{L^{2n+2}}{SU^{2n+1}} + \left(r - q + \frac{\sigma^{2}}{2}\right)T}{\sigma\sqrt{T}} \\ \mu = \frac{r - q - \frac{\sigma^{2}}{2}}{\sigma^{2}}.$$

A knock-in type double barrier asset-at-expiry option has value

$$\left(Se^{-qT}\left(1-\sum_{n=-\infty}^{\infty}\left\{\left(\frac{U^{n}}{L^{n}}\right)^{2(\mu+1)}\left[N\left(a_{1}\right)-N\left(a_{3}\right)\right]-\left(\frac{L^{n+1}}{SU^{n}}\right)^{2(\mu+1)}\left[N\left(a_{5}\right)-N\left(a_{7}\right)\right]\right\}\right).\right)$$

3 Properties of Instrument

Double barrier asset-at-expiry options are options with the asset as payoff at expiry, with lower and upper barriers, where the payoff of the option depends on whether the barriers were touched.

For a knock-out type option, the payoff is the asset, provided both barriers were *not* touched during the life of the option, and zero otherwise.

For a knock-in type option, the payoff is the asset, provided at least one of the barriers *was* touched during the life of the option, and zero otherwise.