Double Barrier Cash-at-Expiry Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Payoff cash amount	K	0^+	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	
In or Out	*indicator*	–	–	"I", "O"

Table 1: Inputs for Double Barrier Cash-at-Expiry Option pricing function

2 Formula

The value of a knock-out type *double barrier cash-at-expiry* option is given by

$$Ke^{-rT}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{U^n}{L^n}\right)^{2\mu}\left[N\left(a_2\right)-N\left(a_4\right)\right]-\left(\frac{L^{n+1}}{SU^n}\right)^{2\mu}\left[N\left(a_6\right)-N\left(a_8\right)\right]\right\},$$

where

$$a_2=\frac{\ln\frac{SU^{2n}}{L^{2n+1}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad a_4=\frac{\ln\frac{SU^{2n-1}}{L^{2n}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$a_6=\frac{\ln\frac{L^{2n+1}}{SU^{2n}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad a_8=\frac{\ln\frac{L^{2n+2}}{SU^{2n+1}}+\left(r-q-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$\mu=\frac{r-q-\frac{\sigma^2}{2}}{\sigma^2}.$$

A knock-in type double barrier cash-at-expiry option has value

$$Ke^{-rT}\left(1-\sum_{n=-\infty}^{\infty}\left\{\left(\frac{U^n}{L^n}\right)^{2\mu}\left[N\left(a_2\right)-N\left(a_4\right)\right]-\left(\frac{L^{n+1}}{SU^n}\right)^{2\mu}\left[N\left(a_6\right)-N\left(a_8\right)\right]\right\}\right).$$

3 Properties of Instrument

Double barrier cash-at-expiry options are options with cash as payoff at expiry, with lower and upper barriers, where the payoff of the option depends on whether the barriers were touched.

For a knock-out type option, the payoff is K provided both barriers were *not* touched during the life of the option, and zero otherwise.

For a knock-in type option, the payoff is K provided at least one of the barriers *was* touched during the life of the option, and zero otherwise.