Double Barrier Cash-at-Touch Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Lower cash payoff amount	K_L	0^+	$+\infty$	
Upper cash payoff amount	K_U	0	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	

Table 1: Inputs for Double Barrier Cash-at-Touch Option pricing function

2 Formula

The value of a *double barrier cash-at-touch* option is given by

$$
\begin{aligned}
&K_U \left\{ \left(\frac{U}{S}\right)^{\mu+\lambda} \left[\sum_{n=-\infty}^{0} \left(\frac{L^{2n}}{U^{2n}}\right)^{\lambda} N(d_3) - \sum_{n=1}^{\infty} \left(\frac{L^{2n}}{U^{2n}}\right)^{\lambda} N(-d_3) \right] \right. \\
&\qquad \left. + \left(\frac{U}{S}\right)^{\mu-\lambda} \left[\sum_{n=-\infty}^{0} \left(\frac{U^{2n}}{L^{2n}}\right)^{\lambda} N(d_4) - \sum_{n=1}^{\infty} \left(\frac{U^{2n}}{L^{2n}}\right)^{\lambda} N(-d_4) \right] \right\} \\
&\quad - K_L \left\{ \left(\frac{L}{S}\right)^{\mu+\lambda} \left[\sum_{n=-\infty}^{-1} \left(\frac{L^{2n}}{U^{2n}}\right)^{\lambda} N(d_1) - \sum_{n=0}^{\infty} \left(\frac{L^{2n}}{U^{2n}}\right)^{\lambda} N(-d_1) \right] \right. \\
&\qquad \left. + \left(\frac{L}{S}\right)^{\mu-\lambda} \left[\sum_{n=-\infty}^{-1} \left(\frac{U^{2n}}{L^{2n}}\right)^{\lambda} N(d_2) - \sum_{n=0}^{\infty} \left(\frac{U^{2n}}{L^{2n}}\right)^{\lambda} N(-d_2) \right] \right\},
\end{aligned}
$$

where

$$
\begin{aligned}
d_1 &= \frac{\ln \frac{SU^{2n}}{L^{2n+1}}}{\sigma\sqrt{T}} - \lambda\sigma\sqrt{T} & d_2 &= \frac{\ln \frac{SU^{2n}}{L^{2n+1}}}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \\
d_3 &= \frac{\ln \frac{SU^{2n-1}}{L^{2n}}}{\sigma\sqrt{T}} - \lambda\sigma\sqrt{T} & d_4 &= \frac{\ln \frac{SU^{2n+1}}{L^{2n+2}}}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \\
\mu &= \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2} & \lambda &= \sqrt{\mu^2 + \frac{2r}{\sigma^2}}.
\end{aligned}
$$

3 Properties of Instrument

Double barrier cash-at-touch options are options with cash as payoff at the moment when either barrier is touched. If the lower barrier was touched first, the immediate payoff is the lower cash payoff amount. If the upper barrier was touched first, the immediate payoff is the upper cash payoff amount.