Double Partial Barrier Early Finish Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Continuous risk-free interest rate till t_1	r_1	0^+	$+\infty$	
Continuous secondary rate till t_1	q_1	0^+	$+\infty$	
Volatility till t_1	σ_1	0^+	$+\infty$	
Time to barrier end	t_1	0^+	$< T_2$	
Continuous risk-free interest rate till T_2	r_2	0^+	$+\infty$	
Continuous secondary rate till T_2	q_2	0^+	$+\infty$	
Volatility till T_2	σ_2	0^+	$+\infty$	
Time to option maturity	T_2	$> t_1$	$+\infty$	
Put or Call	*indicator*	–	–	“P”, “C”
In or Out		–	–	“I”, “O”

Table 1: Inputs for Double Partial Barrier Early Finish Option pricing function

2 Formula

For a knock-out type *double partial barrier early finish* option, the value is

$$
\begin{aligned}
&\phi S e^{-q_2 T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n} \right)^{2(\mu_1+1)} \left[N_2\left(l_1 + a, \phi x_1; \phi\rho\right) - N_2\left(u_1 + a, \phi x_1; \phi\rho\right) \right] \right\} \\
&\quad - \phi S e^{-q_2 T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{S U^n} \right)^{2(\mu_1+1)} \left[N_2\left(l_1 + b, \phi x_3; \phi\rho\right) - N_2\left(u_1 + b, \phi x_3; \phi\rho\right) \right] \right\} \\
&\quad - \phi X e^{-r_2 T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n} \right)^{2\mu_1} \left[N_2\left(l_2 + a, \phi x_2; \phi\rho\right) - N_2\left(u_2 + a, \phi x_2; \phi\rho\right) \right] \right\} \\
&\quad + \phi X e^{-r_2 T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{S U^n} \right)^{2\mu_1} \left[N_2\left(l_2 + b, \phi x_4; \phi\rho\right) - N_2\left(u_2 + b, \phi x_4; \phi\rho\right) \right] \right\},
\end{aligned}
$$

where

$$x_1 = \frac{\ln \frac{SU^{2n}}{XL^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2 \sqrt{T_2}} \qquad x_2 = x_1 - \sigma_2 \sqrt{T_2}$$

$$x_3 = \frac{\ln \frac{L^{2n+2}}{SXU^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2 \sqrt{T_2}} \qquad x_4 = x_3 - \sigma_2 \sqrt{T_2}$$

$$l_1 = \frac{\ln \frac{S}{L} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1 \sqrt{t_1}} \qquad l_2 = l_1 - \sigma_1 \sqrt{t_1}$$

$$u_1 = \frac{\ln \frac{S}{U} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1 \sqrt{t_1}} \qquad u_2 = u_1 - \sigma_1 \sqrt{t_1}$$

$$a = \frac{\ln \frac{U^{2n}}{L^{2n}}}{\sigma_1 \sqrt{t_1}} \qquad b = \frac{\ln \frac{L^{2n+2}}{S^2 U^{2n}}}{\sigma_1 \sqrt{t_1}}$$

$$\rho = \frac{\sigma_1 \sqrt{t_1}}{\sigma_2 \sqrt{T_2}} \qquad \mu_1 = \frac{r_1 - q_1 - \frac{\sigma_1^2}{2}}{\sigma_1^2}.$$

The value of a knock-in type double partial barrier early finish option is equal to a long position in the equivalent vanilla option and a short position in the equivalent knock-out type double partial barrier early finish option.

3 Properties of Instrument

In a standard double barrier option, the barriers are observed for its entire life, from time $t = 0$ to time $t = T_{exp} = T_2$, with the value of the option determined accordingly. In a partial barrier option, the period of observation is a subset of the life of the option. Consider times t_1 such that $0 < t_1 < T_2$.

In an early finish double partial barrier option, the barriers are active for the time frame $0 \leq t \leq t_1$. If either barrier is touched during this time, the option value is changed accordingly. Touching or crossing the barriers during $t_1 < t \leq T_2$ does not affect the value of the option.