Double Partial Barrier Late Start Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Continuous risk-free interest rate till t_1	r_1	0^+	$+\infty$	
Continuous secondary rate till t_1	q_1	0^+	$+\infty$	
Volatility till t_1	σ_1	0^+	$+\infty$	
Time to start of barrier	t_1	0^+	$< T_2$	
Continuous risk-free interest rate till T_2	r_2	0^+	$+\infty$	
Continuous secondary rate till T_2	q_2	0^+	$+\infty$	
Volatility till T_2	σ_2	0^+	$+\infty$	
Time to option maturity	T_2	$> t_1$	$+\infty$	
Put or Call	*indicator*	–	–	"P", "C"
In or Out		–	–	"I", "O"

Table 1: Inputs for Double Partial Barrier Late Start Option pricing function

2 Formula

The value of a knock-out type *double partial barrier late start* ***call*** option is given by

$$\begin{aligned} & Se^{-q_2T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n} \right)^{2(\mu_2+1)} \Big[N_2(l_1, b_1; \rho) - N_2(u_1, b_1; \rho) - N_2(l_1, b_3; \rho) + N_2(u_1, b_3; \rho) \Big] \right\} \\ & - Se^{-q_2T_2+2(\mu_2+1)\alpha t_1} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{SU^n} \right)^{2(\mu_2+1)} \Big[N_2(u_3, b_5; \rho) - N_2(l_3, b_5; \rho) - N_2(u_3, b_7; \rho) + N_2(l_3, b_7; \rho) \Big] \right\} \\ & - Xe^{-r_2T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n} \right)^{2\mu_2} \Big[N_2(l_2, b_2; \rho) - N_2(u_2, b_2; \rho) - N_2(l_2, b_4; \rho) + N_2(u_2, b_4; \rho) \Big] \right\} \\ & + Xe^{-r_2T_2+2\mu_2\alpha t_1} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{SU^n} \right)^{2\mu_2} \Big[N_2(u_4, b_6; \rho) - N_2(l_4, b_6; \rho) - N_2(u_4, b_8; \rho) + N_2(l_4, b_8; \rho) \Big] \right\}, \quad (1) \end{aligned}$$

where

$$b_1 = \frac{\ln \frac{SU^{2n}}{\max(X,L)L^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2\sqrt{T_2}} \qquad b_2 = b_1 - \sigma_2\sqrt{T_2}$$

$$b_3 = \frac{\ln \frac{SU^{2n-1}}{L^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2\sqrt{T_2}} \qquad b_4 = b_3 - \sigma_2\sqrt{T_2}$$

$$b_5 = \frac{\ln \frac{L^{2n+2}}{\max(X,L)SU^{2n}} - \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2 + \beta}{\sigma_2\sqrt{T_2}} \qquad b_6 = b_5 - \sigma_2\sqrt{T_2}$$

$$b_7 = \frac{\ln \frac{L^{2n+2}}{SU^{2n+1}} - \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2 + \beta}{\sigma_2\sqrt{T_2}} \qquad b_8 = b_7 - \sigma_2\sqrt{T_2}$$

$$l_1 = \frac{\ln \frac{S}{L} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1\sqrt{t_1}} \qquad l_2 = l_1 - \sigma_1\sqrt{t_1}$$

$$l_3 = -l_1 + 2\left(\mu_2 + 1\right)\sigma_1\sqrt{t_1} \qquad l_4 = l_3 - \sigma_1\sqrt{t_1}$$

$$u_1 = \frac{\ln \frac{S}{U} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1\sqrt{t_1}} \qquad u_2 = u_1 - \sigma_1\sqrt{t_1}$$

$$u_3 = -u_1 + 2\left(\mu_2 + 1\right)\sigma_1\sqrt{t_1} \qquad u_4 = u_3 - \sigma_1\sqrt{t_1}$$

$$\rho = \frac{\sigma_1\sqrt{t_1}}{\sigma_2\sqrt{T_2}} \qquad \beta = 2\left(\mu_2 + 1\right)\sigma_2^2 T_2$$

$$\mu_1 = \frac{r_1 - q_1 - \frac{\sigma_1^2}{2}}{\sigma_1^2} \qquad \mu_2 = \frac{\left(r_2 - q_2 - \frac{\sigma_2^2}{2}\right) T_2 - \left(r_1 - q_1 - \frac{\sigma_1^2}{2}\right) t_1}{\sigma_2^2 T_2 - \sigma_1^2 t_1}$$

$$\alpha = \left(\mu_2 - \mu_1\right)\sigma_1^2.$$

The value of a knock-out type *double partial barrier late start* ***put*** option is given by

$$\begin{aligned}
&Xe^{-r_2T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{2\mu_2} \left[N_2\left(l_2, c_2; \rho\right) - N_2\left(u_2, c_2; \rho\right) - N_2\left(l_2, c_4; \rho\right) + N_2\left(u_2, c_4; \rho\right)\right] \right\} \\
&- Xe^{-r_2T_2 + 2\mu_2\alpha t_1} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{SU^n}\right)^{2\mu_2} \left[N_2\left(u_4, c_6; \rho\right) - N_2\left(l_4, c_6; \rho\right) - N_2\left(u_4, c_8; \rho\right) + N_2\left(l_4, c_8; \rho\right)\right] \right\} \\
&- Se^{-q_2T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{2(\mu_2+1)} \left[N_2\left(l_1, c_1; \rho\right) - N_2\left(u_1, c_1; \rho\right) - N_2\left(l_1, c_3; \rho\right) + N_2\left(u_1, c_3; \rho\right)\right] \right\} \\
&+ Se^{-q_2T_2 + 2(\mu_2+1)\alpha t_1} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{SU^n}\right)^{2(\mu_2+1)} \left[N_2\left(u_3, c_5; \rho\right) - N_2\left(l_3, c_5; \rho\right) - N_2\left(u_3, c_7; \rho\right) + N_2\left(l_3, c_7; \rho\right)\right] \right\}, \qquad (2)
\end{aligned}$$

where

$$c_1 = \frac{\ln \frac{SU^{2n}}{L^{2n+1}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2\sqrt{T_2}} \qquad c_2 = c_1 - \sigma_2\sqrt{T_2}$$

$$c_3 = \frac{\ln \frac{SU^{2n}}{\min(X,U)L^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2\sqrt{T_2}} \qquad c_4 = c_3 - \sigma_2\sqrt{T_2}$$

$$c_5 = \frac{\ln \frac{L^{2n+2}}{SLU^{2n}} - \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2 + \beta}{\sigma_2\sqrt{T_2}} \qquad c_6 = c_5 - \sigma_2\sqrt{T_2}$$

$$c_7 = \frac{\ln \frac{L^{2n+2}}{\min(X,U)SU^{2n}} - \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2 + \beta}{\sigma_2\sqrt{T_2}} \qquad c_8 = c_7 - \sigma_2\sqrt{T_2}.$$

The above formulae for value of knock-out type double partial barrier late start call and put options can be combined as

$$
\begin{aligned}
&\phi S e^{-q_2 T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{2(\mu_2+1)} \left[N_2(l_1, a_1; \rho) - N_2(u_1, a_1; \rho) - N_2(l_1, a_3; \rho) + N_2(u_1, a_3; \rho) \right] \right\} \\
&- \phi S e^{-q_2 T_2 + 2(\mu_2+1)\alpha t_1} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{SU^n}\right)^{2(\mu_2+1)} \left[N_2(u_3, a_5; \rho) - N_2(l_3, a_5; \rho) - N_2(u_3, a_7; \rho) + N_2(l_3, a_7; \rho) \right] \right\} \\
&- \phi X e^{-r_2 T_2} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{U^n}{L^n}\right)^{2\mu_2} \left[N_2(l_2, a_2; \rho) - N_2(u_2, a_2; \rho) - N_2(l_2, a_4; \rho) + N_2(u_2, a_4; \rho) \right] \right\} \\
&+ \phi X e^{-r_2 T_2 + 2\mu_2 \alpha t_1} \sum_{n=-\infty}^{\infty} \left\{ \left(\frac{L^{n+1}}{SU^n}\right)^{2\mu_2} \left[N_2(u_4, a_6; \rho) - N_2(l_4, a_6; \rho) - N_2(u_4, a_8; \rho) + N_2(l_4, a_8; \rho) \right] \right\},
\end{aligned}
$$

where

$$a_1 = \frac{\ln \frac{SU^{2n}}{\gamma L^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2 \sqrt{T_2}} \qquad a_2 = a_1 - \sigma_2 \sqrt{T_2}$$

$$a_3 = \frac{\ln \frac{SU^{2n}}{\delta L^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2 \sqrt{T_2}} \qquad a_4 = a_3 - \sigma_2 \sqrt{T_2}$$

$$a_5 = \frac{\ln \frac{L^{2n+2}}{\gamma SU^{2n}} - \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2 + \beta}{\sigma_2 \sqrt{T_2}} \qquad a_6 = a_5 - \sigma_2 \sqrt{T_2}$$

$$a_7 = \frac{\ln \frac{L^{2n+2}}{\delta SU^{2n}} - \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2 + \beta}{\sigma_2 \sqrt{T_2}} \qquad a_8 = a_7 - \sigma_2 \sqrt{T_2}$$

$$l_1 = \frac{\ln \frac{S}{L} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1 \sqrt{t_1}} \qquad l_2 = l_1 - \sigma_1 \sqrt{t_1}$$

$$l_3 = -l_1 + 2(\mu_2 + 1)\sigma_1 \sqrt{t_1} \qquad l_4 = l_3 - \sigma_1 \sqrt{t_1}$$

$$u_1 = \frac{\ln \frac{S}{U} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1 \sqrt{t_1}} \qquad u_2 = u_1 - \sigma_1 \sqrt{t_1}$$

$$u_3 = -u_1 + 2(\mu_2 + 1)\sigma_1 \sqrt{t_1} \qquad u_4 = u_3 - \sigma_1 \sqrt{t_1}$$

$$\rho = \frac{\sigma_1 \sqrt{t_1}}{\sigma_2 \sqrt{T_2}} \qquad \beta = 2(\mu_2 + 1)\sigma_2^2 T_2$$

$$\mu_1 = \frac{r_1 - q_1 - \frac{\sigma_1^2}{2}}{\sigma_1^2} \qquad \mu_2 = \frac{\left(r_2 - q_2 - \frac{\sigma_2^2}{2}\right) T_2 - \left(r_1 - q_1 - \frac{\sigma_1^2}{2}\right) t_1}{\sigma_2^2 T_2 - \sigma_1^2 t_1}$$

$$\alpha = (\mu_2 - \mu_1)\sigma_1^2.$$

Option Type	ϕ	γ	δ
Put	-1	L	$\min(X, U)$
Call	1	$\max(X, L)$	U

The value of a knock-in type double partial barrier late start option is equal to a long position in the equivalent vanilla option and a short position in the equivalent knock-out type double partial barrier late start option.

3 Properties of Instrument

In a standard double barrier option, the barriers are observed for its entire life, from time $t = 0$ to time $t = T_{exp} = T_2$, with the value of the option determined accordingly. In a partial barrier option, the period of observation is a subset of the life of the option. Consider times t_1 such that $0 < t_1 < T_2$.

In a late start double partial barrier option, the barriers are active for the time frame $t_1 \leq t \leq T_2$, with two existing approaches to value the option:

1) the option is knocked in or out *only* when either barrier is touched or crossed, or

2) the barriers are regarded as a division of the outcome space (giving an area where the option is 'in' and one where it is 'out'), with evaluation depending on the position of the underlying in the outcome space during the life of the option.

Changes in the underlying during $0 < t < t_1$ do not affect the value of the option.

We consider only case 2).

We give only the valuation method for knock-out type options. As for all barrier options, corresponding knock-in type options can be valued by taking the difference between 'equivalent' vanilla options and matching knock-out type options.