Double Pin Barrier Option

Vector Risk Pty Ltd

April 06, 2017

Version 8.0.7905

1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying price	S	0^+	$+\infty$	
Strike price	X	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Continuous risk-free interest rate till t_1	r_1	0^+	$+\infty$	
Continuous secondary rate till t_1	q_1	0^+	$+\infty$	
Volatility till t_1	σ_1	0^+	$+\infty$	
Time to barrier observation	t_1	0^+	$< T_2$	
Continuous risk-free interest rate till T_2	r_2	0^+	$+\infty$	
Continuous secondary rate till T_2	q_2	0^+	$+\infty$	
Volatility till T_2	σ_2	0^+	$+\infty$	
Time to option maturity	T_2	$> t_1$	$+\infty$	
Put or Call	*indicator*	–	–	"P", "C"
In or Out		–	–	"I", "O"

Table 1: Inputs for Double Pin Barrier Option pricing function

2 Formula

The value of a knock-out type *double pin barrier* option is

$$\phi S e^{-q_2 T_2}\left[N_2\left(l_1, \phi b_1; \phi\rho\right) - N_2\left(u_1, \phi b_1; \phi\rho\right)\right] - \phi X e^{-r_2 T_2}\left[N_2\left(l_2, \phi b_2; \phi\rho\right) - N_2\left(u_2, \phi b_2; \phi\rho\right)\right],$$

where

$$l_1 = \frac{\ln\frac{S}{L} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right)t_1}{\sigma_1\sqrt{t_1}} \qquad l_2 = l_1 - \sigma_1\sqrt{t_1}$$

$$u_1 = \frac{\ln\frac{S}{U} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right)t_1}{\sigma_1\sqrt{t_1}} \qquad u_2 = u_1 - \sigma_1\sqrt{t_1}$$

$$b_1 = \frac{\ln\frac{S}{X} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right)T_2}{\sigma_2\sqrt{T_2}} \qquad b_2 = b_1 - \sigma_2\sqrt{T_2}$$

$$\rho = \frac{\sigma_1\sqrt{t_1}}{\sigma_2\sqrt{T_2}}.$$

ϕ	Option Type
-1	Put
1	Call

The value of a knock-in type double pin barrier option is equal to a long position in the equivalent vanilla option and a short position in the equivalent knock-out type double pin barrier.

For a knock-in type *double pin barrier* option, the value is

$$\begin{aligned}&\phi S e^{-q_2 T_2} N\left(\phi b_1\right) - \phi X e^{-r_2 T_2} N\left(\phi b_2\right) \\ &\quad - \phi S e^{-q_2 T_2}\left[N_2\left(l_1, \phi b_1; \phi\rho\right) - N_2\left(u_1, \phi b_1; \phi\rho\right)\right] + \phi X e^{-r_2 T_2}\left[N_2\left(l_2, \phi b_2; \phi\rho\right) - N_2\left(u_2, \phi b_2; \phi\rho\right)\right].\end{aligned}$$

3 Properties of Instrument

Consider a double pin barrier option, an option with upper and lower barriers that are only applicable at one point during the option lifetime, at time t_1.

For a knock-in type double pin barrier option, the payoff is as for a vanilla option, provided the spot rate is *outside* the upper and lower barriers at time t_1. For a knock-out type double pin barrier option, the payoff is as for a vanilla option provided the spot rate is *between* the barrier levels at time t_1. Knock-in type barrier options are valued by taking the difference between the value of an equivalent vanilla option at maturity and the value of the corresponding knock-out barrier option.