Double Window Barrier Option

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April 06, 2017

Version 8.0.7905

1 Input to function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Lower barrier level	L	0^+	$< U$	
Upper barrier level	U	$> L$	$+\infty$	
Continuous risk-free interest rate until t_1	r_1	0^+	$+\infty$	
Continuous secondary rate until t_1	q_1	0^+	$+\infty$	
Volatility until t_1	σ_1	0^+	$+\infty$	
Time to start of window	t_1	0^+	$< t_2$	
Continuous risk-free interest rate until t_2	r_2	0^+	$+\infty$	
Continuous secondary rate until t_2	q_2	0^+	$+\infty$	
Volatility until t_2	σ_2	0^+	$+\infty$	
Time to end of window	t_2	$> t_1$	$< T_3$	
Continuous risk-free interest rate until T_3	r_3	0^+	$+\infty$	
Continuous secondary rate until T_3	q_3	0^+	$+\infty$	
Volatility until T_3	σ_3	0^+	$+\infty$	
Time to maturity	T_3	$> t_2$	$+\infty$	
Put or Call	*indicator*	–	–	“P”, “C”
In or Out		–	–	“I”, “O”

Table 1: Inputs for Double Window Barrier Option pricing function

2 Formula

The value f_{out} of a *double window barrier* knock-out type option is given by

$$
\begin{aligned}
f_{out} = \phi S e^{-q_3 T_3} \sum_{n=-\infty}^{\infty} \Bigg\{ \left(\frac{U^n}{L^n}\right)^{2(\mu_{12}+1)} \Big[& N_3\left(l_1, c_1, \phi e_1; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \\
- N_3\left(u_1, c_1, \phi e_1; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) - N_3\left(l_1, c_3, \phi e_1; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) & + N_3\left(u_1, c_3, \phi e_1; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \Big] \Bigg\} \\
- \phi S e^{-q_3 T_3 + 2(\mu_{12}+1)\beta\sigma_1^2 t_1} \sum_{n=-\infty}^{\infty} \Bigg\{ \left(\frac{L^{n+1}}{SU^n}\right)^{2(\mu_{12}+1)} \Big[& N_3\left(u_3, d_1, \phi e_3; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \\
- N_3\left(l_3, d_1, \phi e_3; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) - N_3\left(u_3, d_3, \phi e_3; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) & + N_3\left(l_3, d_3, \phi e_3; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \Big] \Bigg\} \\
- \phi X e^{-r_3 T_3} \sum_{n=-\infty}^{\infty} \Bigg\{ \left(\frac{U^n}{L^n}\right)^{2\mu_{12}} \Big[& N_3\left(l_2, c_2, \phi e_2; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \\
- N_3\left(u_2, c_2, \phi e_2; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) - N_3\left(l_2, c_4, \phi e_2; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) & + N_3\left(u_2, c_4, \phi e_2; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \Big] \Bigg\} \\
+ \phi X e^{-r_3 T_3 + 2\mu_{12}\beta\sigma_1^2 t_1} \sum_{n=-\infty}^{\infty} \Bigg\{ \left(\frac{L^{n+1}}{SU^n}\right)^{2\mu_{12}} \Big[& N_3\left(u_4, d_2, \phi e_4; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \\
- N_3\left(l_4, d_2, \phi e_4; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) - N_3\left(u_4, d_4, \phi e_4; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) & + N_3\left(l_4, d_4, \phi e_4; \rho_{12}, \phi\rho_{13}, \phi\rho_{23}\right) \Big] \Bigg\},
\end{aligned}
$$

where

$$
\begin{aligned}
u_1 &= \frac{\ln\frac{S}{U} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1\sqrt{t_1}} & u_2 &= u_1 - \sigma_1\sqrt{t_1} \\
u_3 &= -u_1 + 2\left(\mu_{12}+1\right)\sigma_1\sqrt{t_1} & u_4 &= u_3 - \sigma_1\sqrt{t_1} \\
l_1 &= \frac{\ln\frac{S}{L} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1\sqrt{t_1}} & l_2 &= l_1 - \sigma_1\sqrt{t_1} \\
l_3 &= -l_1 + 2\left(\mu_{12}+1\right)\sigma_1\sqrt{t_1} & l_4 &= l_3 - \sigma_1\sqrt{t_1} \\
c_1 &= \frac{\ln\frac{SU^{2n}}{L^{2n+1}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) t_2}{\sigma_2\sqrt{t_2}} & c_2 &= c_1 - \sigma_2\sqrt{t_2} \\
c_3 &= \frac{\ln\frac{SU^{2n-1}}{L^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) t_2}{\sigma_2\sqrt{t_2}} & c_4 &= c_3 - \sigma_2\sqrt{t_2} \\
d_1 &= \frac{\ln\frac{L^{2n+1}}{SU^{2n}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) t_2 + 2\beta\sigma_1^2 t_1}{\sigma_2\sqrt{t_2}} & d_2 &= d_1 - \sigma_2\sqrt{t_2} \\
d_3 &= \frac{\ln\frac{L^{2n+2}}{SU^{2n+1}} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) t_2 + 2\beta\sigma_1^2 t_1}{\sigma_2\sqrt{t_2}} & d_4 &= d_3 - \sigma_2\sqrt{t_2} \\
e_1 &= \frac{\ln\frac{SU^{2n}}{XL^{2n}} + \left(r_3 - q_3 + \frac{\sigma_3^2}{2}\right) T_3}{\sigma_3\sqrt{T_3}} & e_2 &= e_1 - \sigma_3\sqrt{T_3} \\
e_3 &= \frac{\ln\frac{L^{2n+2}}{SXU^{2n}} + \left(r_3 - q_3 + \frac{\sigma_3^2}{2}\right) T_3 + 2\beta\sigma_1^2 t_1}{\sigma_3\sqrt{T_3}} & e_4 &= e_3 - \sigma_3\sqrt{T_3} \\
\beta &= \mu_{12} - \mu_1 & \mu_1 &= \frac{r_1 - q_1 - \frac{\sigma_1^2}{2}}{\sigma_1^2}
\end{aligned}
$$

$$\mu_{12} = \frac{\left(r_2 - q_2 - \frac{\sigma_2^2}{2}\right) t_2 - \left(r_1 - q_1 - \frac{\sigma_1^2}{2}\right) t_1}{\sigma_2^2 t_2 - \sigma_1^2 t_1} \qquad \rho_{12} = \frac{\sigma_1 \sqrt{t_1}}{\sigma_2 \sqrt{t_2}}$$

$$\rho_{13} = \frac{\sigma_1 \sqrt{t_1}}{\sigma_3 \sqrt{T_3}} \qquad \rho_{23} = \frac{\sigma_2 \sqrt{t_2}}{\sigma_3 \sqrt{T_3}}.$$

ϕ	Option Type
-1	Put
1	Call

The value of a knock-in type double window barrier option is equal to a long position in the equivalent vanilla option and a short position in the equivalent knock-out type double window barrier, that is

$$f_{in} = f_{BSG} - f_{out}.$$

3 Properties of Instrument

A double window barrier option is a double barrier option monitored only within a given time period of the option's life. Let the option run from times t_0 to T_3, with a 'window' existing between times t_1 and t_2, such that $t_0 < t_1 < t_2 < T_3$. If either barrier is touched during $t_1 \to t_2$, the option value is changed accordingly.

Touching or crossing the barrier before t_1, or after t_2, does not affect the value of the option. We define only the value of a knock-out type double window barrier — as with all barriers, corresponding knock-in type options can be valued by taking the difference between 'equivalent' vanilla options and the corresponding knock-out option.