Forward Start Option

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1 Inputs to Function

Description	Symbol	min	max	Reasonable range
Underlying	S	0^{+}	$+\infty$	
Strike price as percentage of underlying on start date	α	0^{+}	$+\infty$	
Continuous risk-free interest rate till t_1	r_1	0^{+}	$+\infty$	
Continuous secondary rate till t_1	q_1	0^{+}	$+\infty$	
Continuous risk-free interest rate till T_2	r_2	0^{+}	$+\infty$	
Continuous secondary rate till T_2	q_2	0^{+}	$+\infty$	
Volatility till T_2	σ_2	0^{+}	$+\infty$	
Time to start date	t_1	0^{+}	< T	
Time to maturity	T_2	$> t_1$	$+\infty$	
Put or Call	indicator	_	—	"P", "C"

Table 1: Inputs for Forward Start Option pricing function

2 Formula

For a *forward start* option, the value is

$$\phi S \left[e^{-q_2 T_2} N \left(\phi d_1 \right) - \alpha e^{-q_1 t_1 - r_{12} (T_2 - t_1)} N \left(\phi d_2 \right) \right],$$

where

$$d_1 = \frac{\ln \frac{1}{\alpha} + \left(r_{12} - q_{12} + \frac{\sigma_{12}^2}{2}\right)(T_2 - t_1)}{\sigma_{12}\sqrt{T_2 - t_1}} \qquad \qquad d_2 = d_1 - \sigma_{12}\sqrt{T_2 - t_1},$$

and r_{12} , q_{12} and σ_{12} are the forward continuous, secondary and volatiliy rates from t_1 to T_2 .

ϕ	Option Type	
-1	Put	
1	Call	



3 Properties of Instrument

Rubinstein (1991) published a formula for valuing a forward start option with time to maturity T, that starts at-themoney, or proportionally in- or out-of-the-money after a known elapsed time t in the future. The strike is set equal to a positive constant α times the asset price S_t at time t.

The payoff of a forward start option at time t is thus

BSG
$$(S_t, \alpha S_t, r_{12}, q_{12}, \sigma_{12}, T-t)$$
,

where BSG() is the value of a Generalised Black-Scholes vanilla option, with uncertain underlying S_t (the value of the underlying at time t), strike αS_t , continuous risk-free rate r_{12} , continuous secondary rate q_{12} , volatility σ_{12} and time to maturity T - t.

Bibliography

Mark Rubinstein. Pay now, choose later. Risk, 4(2):13, February 1991.