# Geometric Average Rate Option

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## 1 Formula

The value of a *forward starting* geometric average rate option is

$$\phi e^{-r_T T}\mathbb{E}\left(G_T\right) N\left(\phi\frac{\mathbb{E}(\ln G_T) - \ln X + \mathrm{Var}(\ln G_T)}{\sqrt{\mathrm{Var}(\ln G_T)}}\right) - \phi X e^{-r_T T} N\left(\phi\frac{\mathbb{E}(\ln G_T) - \ln X}{\sqrt{\mathrm{Var}(\ln G_T)}}\right),$$

where

$$\mathbb{E}\left(\ln G_T\right) = \frac{1}{W}\sum_{i=1}^{n} w_i\left[\ln\mathbb{E}\left(S_{t_i}\right) - \frac{1}{2}\sigma_{t_i}^2 t_i\right] \qquad \mathrm{Var}\left(\ln G_T\right) = \frac{1}{W^2}\sum_{i=1}^{n} w_i\sigma_{t_i}^2 t_i\left(w_i + 2W - 2\sum_{j=1}^{i} w_j\right)$$

$$\mathbb{E}\left(G_T\right) = \exp\left[\mathbb{E}(\ln G_T) + \frac{1}{2}\mathrm{Var}(\ln G_T)\right].$$

| $\phi$ | Option Type |
|---|---|
| -1 | Put |
| 1 | Call |

The value of an *in-progress* geometric average rate option, where the underlying has already been observed at times $t_i$ for $1 \leq i \leq m$, is given by

$$\phi e^{-r_T T}\mathbb{E}\left(G_T\right) N\left(\phi\frac{\mathbb{E}(\ln G_T) - \ln X + \mathrm{Var}(\ln G_T)}{\sqrt{\mathrm{Var}(\ln G_T)}}\right) - \phi X e^{-r_T T} N\left(\phi\frac{\mathbb{E}(\ln G_T) - \ln X}{\sqrt{\mathrm{Var}(\ln G_T)}}\right),$$

where

$$W_H = \sum_{i=1}^{m} w_i \qquad \tilde{W} = \sum_{i=m+1}^{n} w_i \qquad G = \left(\prod_{i=1}^{m} S_{t_i}^{w_i}\right)^{1/W_H}$$

$$\mathbb{E}\left(\ln G_T\right) = \frac{1}{W}\left\{W_H \ln G + \sum_{i=m+1}^{n} w_i\left[\ln\mathbb{E}\left(S_{t_i}\right) - \frac{1}{2}\sigma_{t_i}^2 t_i\right]\right\}$$

$$\mathrm{Var}\left(\ln G_T\right) = \frac{1}{W^2}\sum_{i=m+1}^{n} w_i\sigma_{t_i}^2 t_i\left(w_i + 2\tilde{W} - 2\sum_{j=m+1}^{i} w_j\right)$$

$$\mathbb{E}\left(G_T\right) = \exp\left[\mathbb{E}(\ln G_T) + \frac{1}{2}\mathrm{Var}(\ln G_T)\right].$$

## 2 Properties of Instrument

The payoff for a geometric average rate call option at expiry is given by

$$C_T = (G_T - X)^+ \,, \tag{1}$$

where the weighted geometric mean of the underlying at $n$ pre-defined observation times $t_i$ is given by

$$G_T = \left( \prod_{i=1}^{n} S_{t_i}^{w_i} \right)^{1/W} \,, \tag{2}$$

where $(w_i,\, S_{t_i})$ is the (weight, underlying) pair at $t_i$, and

$$W = \sum_{i=1}^{n} w_i.$$

Similarly, the payoff for a geometric average rate put option at expiry is given by

$$P_T = (X - G_T)^+ \,. \tag{3}$$