Geometric Average Strike Option

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1 Formula

Clewlow and Strickland (1997)[1] define the value of a *forward starting* geometric average strike option as

$$\boxed{\phi\lambda S e^{-q_T T} N(\phi d_1) - \phi e^{-r_T T}\mathbb{E}(G_T) N(\phi d_2),}$$

where

$$d_1 = \frac{\ln(\lambda S) + (r_T - q_T)T - \mathbb{E}(\ln G_T) - \frac{1}{2}\mathrm{Var}(\ln G_T) + \frac{1}{2}\Sigma^2}{\Sigma} \qquad d_2 = d_1 - \Sigma$$

$$\mathbb{E}(\ln G_T) = \frac{1}{W}\sum_{i=1}^{n} w_i\left[\ln\mathbb{E}(S_{t_i}) - \frac{1}{2}\sigma_{t_i}^2 t_i\right] \qquad \mathrm{Var}(\ln G_T) = \frac{1}{W^2}\sum_{i=1}^{n} w_i\sigma_{t_i}^2 t_i\left(w_i + 2W - 2\sum_{j=1}^{i} w_j\right)$$

$$\mathbb{E}(G_T) = \exp\left[\mathbb{E}(\ln G_T) + \frac{1}{2}\mathrm{Var}(\ln G_T)\right] \qquad \Sigma^2 = \sigma_T^2 T + \mathrm{Var}(\ln G_T) - 2\mathrm{Cov}(\ln S_T, \ln G_T)$$

$$\mathrm{Cov}(\ln S_T, \ln G_T) = \frac{1}{W}\sum_{i=1}^{n} w_i\sigma_{t_i}^2 t_i.$$

ϕ	Option Type
-1	Put
1	Call

The value of an *in-progress* geometric average strike option where the underlying has already been observed at times t_i for $1 \leq i \leq m$ is

$$\boxed{\phi\lambda S e^{-qT} N(\phi d_1) - \phi e^{-rT}\mathbb{E}(G_T) N(\phi d_2),}$$

where

$$W_H = \sum_{i=1}^{m} w_i \qquad \tilde{W} = \sum_{i=m+1}^{n} w_i \qquad G = \left(\prod_{i=1}^{m} S_{t_i}^{w_i}\right)^{1/W_H}$$

$$d_1 = \frac{\ln(\lambda S) + (r_T - q_T)T - \mathbb{E}(\ln G_T) - \frac{1}{2}\mathrm{Var}(\ln G_T) + \frac{1}{2}\Sigma^2}{\Sigma} \qquad d_2 = d_1 - \Sigma$$

$$\mathbb{E}(\ln G_T) = \frac{1}{W}\left\{W_H \ln G + \sum_{i=m+1}^{n} w_i\left[\ln\mathbb{E}(S_{t_i}) - \frac{1}{2}\sigma_{t_i}^2 t_i\right]\right\}$$

$$\mathrm{Var}(\ln G_T) = \frac{1}{W^2}\sum_{i=m+1}^{n} w_i\sigma_{t_i}^2 t_i\left(w_i + 2\tilde{W} - 2\sum_{j=m+1}^{i} w_j\right)$$

$$\mathbb{E}(G_T) = \exp\left[\mathbb{E}(\ln G_T) + \frac{1}{2}\mathrm{Var}(\ln G_T)\right] \qquad \Sigma^2 = \sigma_T^2 T + \mathrm{Var}(\ln G_T) - 2\mathrm{Cov}(\ln S_T, \ln G_T)$$

$$\mathrm{Cov}(\ln S_T, \ln G_T) = \frac{1}{W}\sum_{i=m+1}^{n} w_i\sigma_{t_i}^2 t_i.$$

[1]Clewlow and Strickland (1997) p.65–97 *Asian Option by Levy*

2 Properties of Instrument

The payoff of a geometric average rate call option at expiry is given by

$$C_T = (\lambda S_T - G_T)^+ \,, \tag{1}$$

where the weighted geometric mean of the underlying at n pre-defined observation times t_i is given by

$$G_T = \left(\prod_{i=1}^{n} S_{t_i}^{w_i} \right)^{1/W} \,, \tag{2}$$

where $(w_i,\, S_{t_i})$ is the (weight, underlying) pair at time t_i, and

$$W = \sum_{i=1}^{n} w_i .$$

Similarly, the payoff for a geometric average rate put option at expiry is given by

$$P_T = (G_T - \lambda S_T)^+ \,. \tag{3}$$

Bibliography

Les Clewlow and Chris Strickland, editors. *Exotic Options: the State of the Art*. International Thomson Business Press, London, 1997.