Single Barrier Option

Vector Risk Pty Ltd

April 06, 2017

Version 8.0.7905

1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Barrier level	H	0^+	$+\infty$	
Cash amount payoff	K	0^+	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	
Put or Call		–	–	"P", "C"
Up or Down	*indicator*	–	–	"U", "D"
In or Out		–	–	"I", "O"

Table 1: Inputs for Single Barrier Option pricing function

2 Formula

The value of a *single barrier* option is given by Haug (2007) as

1) Down-and-in call $(S > H)$ Payoff: $\max(S_T - X, 0)$ if $S_t \le H$ for some $0 \le t \le T$, K at expiration otherwise.
   Value: $(X > H)$: $C + E$ $\quad \eta = 1, \quad \phi = 1$
   Value: $(X < H)$: $A - B + D + E$ $\quad \eta = 1, \quad \phi = 1$

2) Up-and-in call $(S < H)$ Payoff: $\max(S_T - X, 0)$ if $S_t \ge H$ for some $0 \le t \le T$, K at expiration otherwise.
   Value: $(X > H)$: $A + E$ $\quad \eta = -1, \quad \phi = 1$
   Value: $(X < H)$: $B - C + D + E$ $\quad \eta = -1, \quad \phi = 1$

3) Down-and-in put $(S > H)$ Payoff: $\max(X - S_T, 0)$ if $S_t \le H$ for some $0 \le t \le T$, K at expiration otherwise.
   Value: $(X > H)$: $B - C + D + E$ $\quad \eta = 1, \quad \phi = -1$
   Value: $(X < H)$: $A + E$ $\quad \eta = 1, \quad \phi = -1$

4) Up-and-in put $(S < H)$ Payoff: $\max(X - S_T, 0)$ if $S_t \ge H$ for some $0 \le t \le T$, K at expiration otherwise.
   Value: $(X > H)$: $A - B + D + E$ $\quad \eta = -1, \quad \phi = -1$
   Value: $(X < H)$: $C + E$ $\quad \eta = -1, \quad \phi = -1$

5) Down-and-out call $(S > H)$ Payoff: $\max(S_T - X, 0)$ if $S_t > H$ for all $0 \le t \le T$, K at touch otherwise.
   Value: $(X > H)$: $A - C + F$ $\quad \eta = 1, \quad \phi = 1$
   Value: $(X < H)$: $B - D + F$ $\quad \eta = 1, \quad \phi = 1$

6) Up-and-out call $(S < H)$ Payoff: $\max(S_T - X, 0)$ if $S_t < H$ for all $0 \leq t \leq T$, K at touch otherwise.
Value: $(X > H)$: F $\quad \eta = -1, \quad \phi = 1$
Value: $(X < H)$: $A - B + C - D + F$ $\quad \eta = -1, \quad \phi = 1$

7) Down-and-out put $(S > H)$ Payoff: $\max(X - S_T, 0)$ if $S_t > H$ for all $0 \leq t \leq T$, K at touch otherwise.
Value: $(X > H)$: $A - B + C - D + F$ $\quad \eta = 1, \quad \phi = -1$
Value: $(X < H)$: F $\quad \eta = 1, \quad \phi = -1$

8) Up-and-out put $(S < H)$ Payoff: $\max(X - S_T, 0)$ if $S_t > H$ for all $0 \leq t \leq T$, K at touch otherwise.
Value: $(X > H)$: $B - D + F$ $\quad \eta = -1, \quad \phi = -1$
Value: $(X < H)$: $A - C + F$ $\quad \eta = -1, \quad \phi = -1$

where

$$
\begin{aligned}
A &= \phi S e^{-qT} N(\phi d_1) - \phi X e^{-rT} N(\phi d_2) \\
B &= \phi S e^{-qT} N(\phi h_1) - \phi X e^{-rT} N(\phi h_2) \\
C &= \phi S e^{-qT} \left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_1) - \phi X e^{-rT} \left(\frac{H}{S}\right)^{2\mu} N(\eta y_2) \\
D &= \phi S e^{-qT} \left(\frac{H}{S}\right)^{2(\mu+1)} N(\eta y_3) - \phi X e^{-rT} \left(\frac{H}{S}\right)^{2\mu} N(\eta y_4) \\
E &= K e^{-rT} \left[N(\eta h_2) - \left(\frac{H}{S}\right)^{2\mu} N(\eta y_4) \right] \\
F &= K \left[\left(\frac{H}{S}\right)^{\mu+\lambda} N(\eta z) + \left(\frac{H}{S}\right)^{\mu-\lambda} N\left(\eta z - 2\eta\lambda\sigma\sqrt{T}\right) \right],
\end{aligned}
$$

and

$$
\begin{aligned}
d_1 &= \frac{\ln \frac{S}{X}}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} & d_2 &= d_1 - \sigma\sqrt{T} \\
h_1 &= \frac{\ln \frac{S}{H}}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} & h_2 &= h_1 - \sigma\sqrt{T} \\
y_1 &= \frac{\ln \frac{H^2}{SX}}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} & y_2 &= y_1 - \sigma\sqrt{T} \\
y_3 &= \frac{\ln \frac{H}{S}}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} & y_4 &= y_3 - \sigma\sqrt{T} \\
\mu &= \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2} & \lambda &= \sqrt{\mu^2 + \frac{2r}{\sigma^2}} \\
z &= \frac{\ln \frac{H}{S}}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}.
\end{aligned}
$$

3 Properties of Instrument

Merton (1973) and Reiner and Rubinstein (1991) have developed the formulae for pricing standard single barrier options[1] — options that have a payoff contingent on whether the barrier is hit during its life.

For a knock-out type option, the payoff is vanilla provided the barrier is *not* touched during the life of the option, and the rebate amount otherwise.

For a knock-in type option, the payoff is vanilla provided the barrier *is* touched during the life of the option, and the rebate amount otherwise.

[1] Haug (2007) p.152 4.17.1 *Standard Single Barrier Options*

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas.* McGraw Hill, New York, 2nd edition, 2007.

Robert Cox Merton. Theory of rational option pricing. *Bell Journal of Economics and Management Science*, 4(1): 141–183, Spring 1973.

Eric Reiner and Mark Rubinstein. Breaking down the barriers. *Risk*, 4(8):28–35, September 1991.