Single Barrier Asset-at-Touch Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Barrier level	H	0^+	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	
Up or Down	*indicator*	–	–	"U", "D"

Table 1: Inputs for Single Barrier Asset-at-Touch Option pricing function

2 Formula

The value of a *single barrier asset-at-touch* option is given by[1]

1) Down ($S > H$)
Payoff: $S_t(H)$ at touch if $S_t \leq H$ for some $0 \leq t \leq T$, zero otherwise.
Value: A_5 $\eta = 1$

2) Up ($S < H$)
Payoff: $S_t(H)$ at touch if $S_t \geq H$ for some $0 \leq t \leq T$, zero otherwise.
Value: A_5 $\eta = -1$

where

$$A_5 = H\left[\left(\frac{H}{S}\right)^{\mu+\lambda} N(\eta z) + \left(\frac{H}{S}\right)^{\mu-\lambda} N\left(\eta z - 2\eta\lambda\sigma\sqrt{T}\right)\right]$$

$$z = \frac{\ln\frac{H}{S}}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad \mu = \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2} \qquad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}.$$

3 Properties of Instrument

Reiner and Rubinstein (1991) introduced a set of formulae that can value single barrier asset-at-touch options. Single barrier asset-at-touch options are options with asset as payoff at the time the barrier is touched. If the barrier is not touched, then the payoff is zero.

[1] Haug (2007) p.176 4.19.5 *Binary Barrier Options*

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas*. McGraw Hill, New York, 2nd edition, 2007.

Eric Reiner and Mark Rubinstein. Unscrambling the binary code. *Risk*, 4(9):75–83, October 1991.