Single Barrier Cash-at-Expiry Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Barrier level	H	0^+	$+\infty$	
Cash amount payoff	K	0^+	$+\infty$	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	
Up or Down	*indicator*	–	–	"U", "D"
In or Out		–	–	"I", "O"

Table 1: Inputs for Single Barrier Cash-at-Expiry Option pricing function

2 Formula

The value of a *single barrier cash-at-expiry* option is given by[1]

1) Down-and-in ($S > H$)
   Payoff: K at expiration if $S_t \leq H$ for some $0 \leq t \leq T$, zero otherwise.
   Value: $B_2 + B_4$ $\quad \eta = 1, \quad \phi = -1$

2) Up-and-in ($S < H$)
   Payoff: K at expiration if $S_t \geq H$ for some $0 \leq t \leq T$, zero otherwise.
   Value: $B_2 + B_4$ $\quad \eta = -1, \quad \phi = 1$

3) Down-and-out ($S > H$)
   Payoff: K at expiration if $S_t > H$ for all $0 \leq t \leq T$, zero otherwise.
   Value: $B_2 - B_4$ $\quad \eta = 1, \quad \phi = 1$

4) Up-and-out ($S < H$)
   Payoff: K at expiration if $S_t < H$ for all $0 \leq t \leq T$, zero otherwise.
   Value: $B_2 - B_4$ $\quad \eta = -1, \quad \phi = -1$

[1] Haug (2007) p.176 4.19.5 *Binary Barrier Options*

where

$$B_2 = Ke^{-rT} N\left(\phi h_2\right) \qquad B_4 = Ke^{-rT}\left(\frac{H}{S}\right)^{2\mu} N\left(\eta y_4\right)$$

$$h_2 = \frac{\ln \frac{S}{H}}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T} \qquad y_4 = \frac{\ln \frac{H}{S}}{\sigma\sqrt{T}} + \mu\sigma\sqrt{T}$$

$$\mu = \frac{r - q - \frac{\sigma^2}{2}}{\sigma^2},$$

and

ξ	Barrier Type
-1	In
1	Out

3 Properties of Instrument

Reiner and Rubinstein (1991) introduced a set of formulae that can value single barrier cash-at-expiry options. Single barrier cash-at-expiry options are options with a cash amount as payoff at expiry, with a single barrier, so that the option payoff is dependent on whether the barrier is touched.

For a knock-out type option, the payoff is K provided the barrier is *not* touched during the life of the option, and zero otherwise.

For a knock-in type option, the payoff is K provided the barrier *is* touched during the life of the option, and zero otherwise.

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas*. McGraw Hill, New York, 2nd edition, 2007.

Eric Reiner and Mark Rubinstein. Unscrambling the binary code. *Risk*, 4(9):75–83, October 1991.