Single Barrier Cash-at-Touch Option

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1 Input to Function

Description	Symbol	min	max	$Reasonable\ range$
Underlying	S	0^{+}	$+\infty$	
Barrier level	H	0^{+}	$+\infty$	
Cash amount payoff	K	0^{+}	$+\infty$	
Continuous risk-free interest rate	r	0^{+}	$+\infty$	
Continuous secondary rate	q	0^{+}	$+\infty$	
Volatility	σ	0^{+}	$+\infty$	
Time to maturity	T	0^{+}	$+\infty$	
Up or Down	indicator	_	-	"U", "D"

Table 1: Inputs for Single Barrier Cash-at-Touch Option pricing function

2 Formula

The value of a single barrier cash-at-touch option is given by 1

1) Down (S > H)Payoff: K at touch if $S_t \le H$ for some $0 \le t \le T$, zero otherwise. Value: A_5

 $\eta = 1$

 $\eta = -1$

2) Up (S < H)

Payoff: K at touch if $S_t \ge H$ for some $0 \le t \le T$, zero otherwise. Value: A_5

where

$$A_{5} = K \left[\left(\frac{H}{S} \right)^{\mu + \lambda} N(\eta z) + \left(\frac{H}{S} \right)^{\mu - \lambda} N\left(\eta z - 2\eta \lambda \sigma \sqrt{T} \right) \right]$$
$$z = \frac{\ln \frac{H}{S}}{\sigma \sqrt{T}} + \lambda \sigma \sqrt{T} \qquad \mu = \frac{r - q - \frac{\sigma^{2}}{2}}{\sigma^{2}} \qquad \lambda = \sqrt{\mu^{2} + \frac{2r}{\sigma^{2}}}.$$

3 Properties of Instrument

Reiner and Rubinstein (1991) introduced a set of formulae that can value single barrier cash-at-touch options. Single barrier cash-at-touch options are options with payoff K at the time the barrier is touched. If the barrier is not touched, the payoff is zero.



¹Haug (2007) p.176 4.19.5 Binary Barrier Options

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas*. McGraw Hill, New York, 2nd edition, 2007. Eric Reiner and Mark Rubinstein. Unscrambling the binary code. *Risk*, 4(9):75–83, October 1991.