Single Partial Barrier Early Finish Option

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1 Input to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying price	S	0^+	$+\infty$	
Strike price	X	0^+	$+\infty$	
Barrier level	H	0^+	$+\infty$	
Continuous risk-free interest rate until t_1	r_1	0^+	$+\infty$	
Continuous secondary rate until t_1	q_1	0^+	$+\infty$	
Volatility until t_1	σ_1	0^+	$+\infty$	
Time to barrier end	t_1	0^+	$< T_2$	
Continuous risk-free interest rate until T_2	r_2	0^+	$+\infty$	
Continuous secondary rate until T_2	q_2	0^+	$+\infty$	
Volatility until T_2	σ_2	0^+	$+\infty$	
Time to option maturity	T_2	$> t_1$	$+\infty$	
Put or Call		–	–	"P", "C"
Up or Down	*indicator*	–	–	"U", "D"
In or Out		–	–	"I", "O"

Table 1: Inputs for Single Partial Barrier Early Finish Option pricing function

2 Formula

The value of a *single partial barrier early finish* option is given by

$$\boxed{\begin{aligned}&\phi S e^{-q_2 T_2}\left[N_2\left(\xi\eta h_1, \phi b_1; \xi\eta\phi\rho\right) - \xi\left(\frac{H}{S}\right)^{2(\mu_1+1)} N_2\left(\eta f_1, \phi g_1; \eta\phi\rho\right)\right]\\ &\qquad - \phi X e^{-r_2 T_2}\left[N_2\left(\xi\eta h_2, \phi b_2; \xi\eta\phi\rho\right) - \xi\left(\frac{H}{S}\right)^{2\mu_1} N_2\left(\eta f_2, \phi g_2; \eta\phi\rho\right)\right],\end{aligned}}$$

where

$$h_1 = \frac{\ln\frac{S}{H} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right)t_1}{\sigma_1\sqrt{t_1}} \qquad h_2 = h_1 - \sigma_1\sqrt{t_1}$$

$$f_1 = \frac{\ln\frac{H}{S} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right)t_1}{\sigma_1\sqrt{t_1}} \qquad f_2 = f_1 - \sigma_1\sqrt{t_1}$$

$$g_1 = \frac{\ln\frac{H^2}{SX} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right)T_2}{\sigma_2\sqrt{T_2}} \qquad g_2 = g_1 - \sigma_2\sqrt{T_2}$$

$$b_1 = \frac{\ln \frac{S}{X} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) T_2}{\sigma_2 \sqrt{T_2}} \qquad b_2 = b_1 - \sigma_2 \sqrt{T_2}$$

$$\mu_1 = \frac{r_1 - q_1 - \frac{\sigma_1^2}{2}}{\sigma_1^2} \qquad \rho = \frac{\sigma_1 \sqrt{t_1}}{\sigma_2 \sqrt{T_2}},$$

and

ϕ	Option Type	η	Barrier Direction	ξ	Barrier Type
-1	Put	-1	Up	-1	In
1	Call	1	Down	1	Out

3 Properties of Instrument

In a standard single barrier option, the barrier is observed for its entire life, from time $t = 0$ to time $t = T_{exp} = T_2$, with the value of the option determined accordingly. In a partial barrier option, the period of observation is a subset of the life of the option. Consider times t_1 such that $0 < t_1 < T_2$.

In an early finish single partial barrier option, the barrier is active for the time frame $0 \leq t \leq t_1$. If the barrier is touched during this time, the option value is changed accordingly. Touching or crossing the barrier during $t_1 < t \leq T_2$ does not affect the value of the option.

Formulae for pricing this type of option were originally published by Heynen and Kat (1994)[1].

Bibliography

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas.* McGraw Hill, New York, 2nd edition, 2007.

Ronald C. Heynen and Harry M. Kat. Partial barrier options. *The Journal of Financial Engineering*, 3(3/4):253–274, September/December 1994.

[1] Haug (2007) p.160 4.17.4 *Partial-Time Single-Asset Barrier Options*