# Single Window Barrier Option

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April 13, 2017

Version 8.0.7970

## 1 Input to function

| *Description* | *Symbol* | *min* | *max* | *Reasonable range* |
|---|---|---|---|---|
| Underlying | $S$ | $0^+$ | $+\infty$ | |
| Strike | $X$ | $0^+$ | $+\infty$ | |
| Barrier | $H$ | $0^+$ | $+\infty$ | |
| Continuous risk-free interest rate until $t_1$ | $r_1$ | $0^+$ | $+\infty$ | |
| Continuous secondary rate until $t_1$ | $q_1$ | $0^+$ | $+\infty$ | |
| Volatility until $t_1$ | $\sigma_1$ | $0^+$ | $+\infty$ | |
| Time to start of window | $t_1$ | $0^+$ | $< t_2$ | |
| Continuous risk-free interest rate until $t_2$ | $r_2$ | $0^+$ | $+\infty$ | |
| Continuous secondary rate until $t_2$ | $q_2$ | $0^+$ | $+\infty$ | |
| Volatility until $t_2$ | $\sigma_2$ | $0^+$ | $+\infty$ | |
| Time to end of window | $t_2$ | $> t_1$ | $< T_3$ | |
| Continuous risk-free interest rate until $T_3$ | $r_3$ | $0^+$ | $+\infty$ | |
| Continuous secondary rate until $T_3$ | $q_3$ | $0^+$ | $+\infty$ | |
| Volatility until $T_3$ | $\sigma_3$ | $0^+$ | $+\infty$ | |
| Time to maturity | $T_3$ | $> t_2$ | $+\infty$ | |
| Put or Call | | – | – | "P", "C" |
| Up or Down | *indicator* | – | – | "U", "D" |
| In or Out | | – | – | "I", "O" |

Table 1: Inputs for Single Window Barrier Option

## 2 Formula

The value of a *Single Window Barrier* knock-out type option is given by

$$\boxed{\begin{aligned}&\phi S e^{-q_3 T_3}\left[N_3\left(\eta x_1, \eta c_1, \phi d_1; \rho_{12}, \eta\phi\rho_{13}, \eta\phi\rho_{23}\right) - \left(\frac{H}{S}e^{\beta t_1}\right)^{2(\mu_2+1)} N_3\left(-\eta x_3, \eta c_3, \phi d_3; -\rho_{12}, -\eta\phi\rho_{13}, \eta\phi\rho_{23}\right)\right]\\ &-\phi X e^{-r_3 T_3}\left[N_3\left(\eta x_2, \eta c_2, \phi d_2; \rho_{12}, \eta\phi\rho_{13}, \eta\phi\rho_{23}\right) - \left(\frac{H}{S}e^{\beta t_1}\right)^{2\mu_2} N_3\left(-\eta x_4, \eta c_4, \phi d_4; -\rho_{12}, -\eta\phi\rho_{13}, \eta\phi\rho_{23}\right)\right],\end{aligned}}$$

where

| $\phi$ | Option Type | $\eta$ | Barrier Direction |
|---|---|---|---|
| -1 | Put | -1 | Up |
| 1 | Call | 1 | Down |

and

$$x_1 = \frac{\ln \frac{S}{H} + \left(r_1 - q_1 + \frac{\sigma_1^2}{2}\right) t_1}{\sigma_1 \sqrt{t_1}} \qquad x_2 = x_1 - \sigma_1 \sqrt{t_1}$$

$$x_3 = x_1 + 2 \frac{\ln \frac{H}{S} + \beta}{\sigma_1 \sqrt{t_1}} \qquad x_4 = x_3 - \sigma_1 \sqrt{t_1}$$

$$c_1 = \frac{\ln \frac{S}{H} + \left(r_2 - q_2 + \frac{\sigma_2^2}{2}\right) t_2}{\sigma_2 \sqrt{t_2}} \qquad c_2 = c_1 - \sigma_2 \sqrt{t_2}$$

$$c_3 = c_1 + 2 \frac{\ln \frac{H}{S} + \beta}{\sigma_2 \sqrt{t_2}} \qquad c_4 = c_3 - \sigma_2 \sqrt{t_2}$$

$$d_1 = \frac{\ln \frac{S}{X} + \left(r_3 - q_3 + \frac{\sigma_3^2}{2}\right) T_3}{\sigma_3 \sqrt{T_3}} \qquad d_2 = d_1 - \sigma_3 \sqrt{T_3}$$

$$d_3 = d_1 + 2 \frac{\ln \frac{H}{S} + \beta}{\sigma_3 \sqrt{T_3}} \qquad d_4 = d_3 - \sigma_3 \sqrt{T_3}$$

$$\rho_{12} = \frac{\sigma_1 \sqrt{t_1}}{\sigma_2 \sqrt{t_2}} \qquad \rho_{13} = \frac{\sigma_1 \sqrt{t_1}}{\sigma_3 \sqrt{T_3}}$$

$$\rho_{23} = \frac{\sigma_2 \sqrt{t_2}}{\sigma_3 \sqrt{T_3}} \qquad \beta = (\mu_2 - \mu_1)\, \sigma_1^2 t_1$$

$$\mu_1 = \frac{r_1 - q_1 - \frac{\sigma_1^2}{2}}{\sigma_1^2} \qquad \mu_2 = \frac{\left(r_2 - q_2 - \frac{\sigma_2^2}{2}\right) t_2 - \mu_1 \sigma_1^2 t_1}{\sigma_2^2 t_2 - \sigma_1^2 t_1}.$$

## 3 Properties of Instrument

A single window barrier option is a single barrier option monitored only within a specified segment of the option's life. Let the option run from times $t_0$ to $T_3$, with a 'window' existing between times $t_1$ and $t_2$, such that $t_0 < t_1 < t_2 < T_3$. If the barrier is touched or crossed during $t_1 \to t_2$, the option is valued accordingly. If equality holds, the option ceases to be a window option, becoming either partial barrier or full barrier options.

Touching or crossing the barrier before $t_1$ or after $t_2$ does not affect the value of the option. We define only the value of a knock-out type single window barrier — as with all barriers, corresponding knock-in type options can be valued by taking the difference between corresponding vanilla and knock-out option.