Variable Purchase Option

Vector Risk Pty Ltd

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1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Underlying	S	0^+	$+\infty$	
Strike	X	0^+	$+\infty$	
Lower level	L	0^+	$\leq U$	
Upper level	U	$\geq L$	$+\infty$	
Discount	D	0^+	< 1	
Continuous risk-free interest rate	r	0^+	$+\infty$	
Continuous secondary rate	q	0^+	$+\infty$	
Volatility	σ	0^+	$+\infty$	
Time to maturity	T	0^+	$+\infty$	

Table 1: Inputs for Variable Purchase Option pricing function

2 Formula

Haug (2007)[1] states that *variable purchase* options can be valued using the Handley (2000) formula. For efficient purpose, our Risk Engine uses the following formula instead.

$$\boxed{N_{\min} S e^{-qT} N(d_1) + N_{\max} S e^{-qT}\left[N(d_5) - N(d_3)\right] + \frac{X}{1-D} e^{-rT}\left[N(d_4) - N(d_2)\right] - X e^{-rT} N(d_6),}$$

where

$$d_1 = \frac{\ln\frac{S}{U} + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

$$d_3 = \frac{\ln\frac{S}{L} + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad d_4 = d_3 - \sigma\sqrt{T}$$

$$d_5 = \frac{\ln\frac{S}{L(1-D)} + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad d_6 = d_5 - \sigma\sqrt{T}$$

$$N_{\min} = \frac{X}{U(1-D)} \qquad N_{\max} = \frac{X}{L(1-D)}.$$

[1] Haug (2007) p.112, 4.1 *Variable Purchase Options*

3 Properties of Instrument

A variable purchase option is a call option on a stochastic number of underlying shares. The actual number of underlying shares is calculated at *maturity* in accordance with a formula which is specified at the *time of issue*. The number of underlying shares at maturity is

$$N = \frac{X}{S_T(1-D)},$$

with S_T representing the value of the underlying at expiry.

The payoff of the option at maturity is

$$\max\left(NS_T - X, 0\right).$$

Further, the number of underlying shares is often subject to a minimum and maximum. The number of underlying shares is

$$\begin{cases} N_{\max} = \dfrac{X}{L(1-D)}; & S_T < L, \\ \dfrac{X}{S_T(1-D)}; & L \leq S_T \leq U, \\ N_{\min} = \dfrac{X}{U(1-D)}; & S_T > U. \end{cases}$$

Table 2 illustrates the payoff for a variable purchase option

Condition	Payoff
$S_T < L(1-D)$	0
$L(1-D) \leq S_T < L$	$\frac{XS_T}{L(1-D)} - X$
$L \leq S_T \leq U$	$\frac{D}{1-D}X$
$S_T > U$	$\frac{XS_T}{U(1-D)} - X$

Table 2: Payoff at maturity for variable purchase option

Bibliography

John Christopher Handley. Variable purchase option. *Review of Derivatives Research*, 4(3):219–230, October 2000.

Espen Gaarder Haug. *The Complete Guide To Option Pricing Formulas*. McGraw Hill, New York, 2nd edition, 2007.

Glossary

Risk Engine The Vector Risk market risk and credit risk system.