FX American Option Product Specification

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Chapter 1

FX American Option

1.1 Instrument Properties

An FX American option is an American option with the **cross currency** as the underlying. At anytime before the **maturity date**, the holder of the option has the right to **early exercise** the option for an immediate payoff and the option will cease to exist. If an FX American **call option** (**put option**) is exercised, the holder buys (sells) the **cross currency amount**, denoted by N_c, at the predetermined **strike rate** X. If we denote the exercise time of the option by τ, and the **primary currency** per **cross currency** exchange rate by S_τ, the payoff by exercising an FX American option is illustrated in Table 1.1.

Option Type	Payoff (**primary currency**)
Call	$N_\mathrm{c}\left(S_\tau - X\right)$
Put	$N_\mathrm{c}\left(X - S_\tau\right)$

Table 1.1: Payoff at time of exercising for FX American option

1.2 Definitions

In this section, we define terms that are specific to FX American options.

call option gives the holder the right, but not the obligation, to buy the **cross currency** at the **strike rate** on or before the **maturity date**.

cross currency is the currency nominated as the underlying asset.

cross currency amount is the deal amount in **cross currency** that will be exchanged if the option is exercised.

early exercise means at anytime before the **maturity date**, the holder of the option decides to exercise the option for an immediate payoff.

maturity date is the date the option expires.

primary currency is the currency the deal is quoted in.

put option gives the holder the right, but not the obligation, to sell the **cross currency** at the **strike rate** on or before the **maturity date**.

strike rate is the agreed exchange rate between **primary currency** and **cross currency** if the option is exercised, quoted in **primary currency** per **cross currency**.

1.3 Representations

In the Risk Engine, products are specified by *representations*. In this section, we provide the representations of FX American options.

1.3.1 Default Representation

The *Default* representation consists of the mandatory trade fields in Table 1.2 with their restrictions in Table 1.3.

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CurrencyAmount	The deal amount in *Currency*	double	N_p
CrossCurrencyAmount	The deal amount in *CrossCurrency*, *i.e.*, the **cross currency amount**	double	N_c
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
BoughtSold	Bought or sold the option	string	BS

Table 1.2: Mandatory trade fields for the Default representation of the FX American Option

Field	*Restriction*
CrossCurrency	$c \neq p$
CurrencyAmount	$N_p > 0$
CrossCurrencyAmount	$N_c > 0$
PutCall	Put, Call, P, C
BoughtSold	Bought, Sold, B, S

Table 1.3: Trade field restrictions for the Default representation of the FX American Option

1.3.1.1 Required Curves

The following curves are required by an FX American option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),

- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),

- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),

- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and

- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

1.3.2 Strike Representation

The *Strike* representation consists of the mandatory trade fields in Table 1.4 with their restrictions in Table 1.5.

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CrossCurrencyAmount	The deal amount in *CrossCurrency*, *i.e.*, the **cross currency amount**	double	N_c
Strike	The **strike rate** as *Currency/CrossCurrency*	double	X
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
BoughtSold	Bought or sold the option	string	BS

Table 1.4: Mandatory trade fields for the Strike representation of the FX American Option

Field	Restriction
CrossCurrency	$c \neq p$
CrossCurrencyAmount	$N_c > 0$
Strike	$X > 0$
PutCall	Put, Call, P, C
BoughtSold	Bought, Sold, B, S

Table 1.5: Trade field restrictions for the Strike representation of the FX American Option

1.3.2.1   Required Curves

The following curves are required by an FX American option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),

- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),

- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),

- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and

- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

1.4   Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX American option in Base Currency is given by the *FX American option pricing function*[1],

$$\text{FXAmerican}\left(E_p, E_c, X, N_c, r_p, r_c, \sigma, T, \text{indicator}\right), \tag{1.1}$$

where

- E_p is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,

- E_c is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,

- X is the **strike rate** in units of **primary currency** per **cross currency**,

- N_c is the **cross currency amount**,

- r_p is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,

- r_c is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,

- σ is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,

- T is the time in years from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, and

- indicator contains the put/call and bought/sold information.

If the Valuation Date is greater than the **maturity date**, then the FX American option has expired and thus has a value of zero.

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[1]See FX American Option Pricing for details (p.8 of this document).

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1.4.1 Representation Reduction

Equation (1.1) is only defined for the Strike representation. If the trade is specified by other representations, we need to reduce it to the Strike representation.

1.4.1.1 Default Representation

For the Default representation, the **strike rate**, X, is the ratio of CurrencyAmount and CrossCurrencyAmount, given by

$$X = \frac{N_{\text{p}}}{N_{\text{c}}}. \tag{1.2}$$

1.5 Examples

This section provides some deal examples of FX American option.

Example 1.1. An FX American option in Default representation:
- Currency: AUD
- CrossCurrency: GBP
- CurrencyAmount: 100,000,000
- CrossCurrencyAmount: 60,000,000
- MaturityDate: 2013-11-15
- PutCall: Put
- BoughtSold: Bought

Using equation (1.2), the **strike rate** of the option is

$$X = \frac{N_{\text{p}}}{N_{\text{c}}} = \frac{100,000,000}{60,000,000} = 1.6667.$$

At any time before the **maturity date**, 2013-11-15, the holder can exercise the option. Say on 2013-10-15, the AUD/GBP exchange rate is 1.6515 and the holder exercises the option. The payoff of the option is

$$N_{\text{c}}\left(X - S_\tau\right) = 60,000,000 \times (1.6667 - 1.6515) = \$910,000 \text{ AUD}.$$

Example 1.2. An FX American option in Strike representation:
- Currency: JPY
- CrossCurrency: USD
- CrossCurrencyAmount: 100,000,000
- Strike: 100.2
- MaturityDate: 2013-11-15
- PutCall: Call
- BoughtSold: Bought

At any time before the **maturity date**, 2013-11-15, the holder can exercise the option. Say on 2013-10-15, the JPY/USD exchange rate is 102.5, and the holder exercises the option. The payoff of the option is

$$N_{\text{c}}\left(S_\tau - X\right) = 100,000,000 \times (102.5 - 100.2) = \$230,000,000 \text{ JPY}.$$

Chapter 2

FX American Option Pricing

2.1   Inputs to Function

Description	Symbol	min	max	Reasonable range
Spot rate of primary currency	E_p	0^+	$+\infty$	
Spot rate of cross currency	E_c	0^+	$+\infty$	
Strike rate as primary currency/cross currency	X	0^+	$+\infty$	
Cross currency amount	N_c	0^+	$+\infty$	
Continuous zero rate of primary currency	r_p	0^+	$+\infty$	
Continuous zero rate of cross currency	r_c	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies	σ	0^+	$+\infty$	
Time from value date to maturity in years	T	0^+	$+\infty$	
Put or Call on cross currency	indicator	$-$	$-$	"P", "C"
Bought or Sold		$-$	$-$	"B", "S"

Table 2.1: Inputs for FX American Option pricing function

2.2   Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_\mathrm{c}}{E_\mathrm{p}}.$$

We can value an FX American option by calling the *American option pricing function*[1] with appropriate inputs. The value of an FX American option in Base Currency is

$$N_\mathrm{c} \times E_\mathrm{p} \times \mathbb{I}_\mathrm{BS} \times \mathrm{American}\left(S, X, r_\mathrm{p}, r_\mathrm{c}, \sigma, T, \mathrm{indicator}\right),$$

where

$$\mathbb{I}_\mathrm{BS} = \begin{cases} 1, & \text{if indicator is 'B',} \\ -1, & \text{if indicator is 'S'.} \end{cases}$$

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[1]See pricing specification *American Option* for details.

Glossary

Base Currency The currency that the risk engine is configured to return values in.

Reserve Currency The currency that all cross currency basis is benchmarked against.

Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.