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FX Double Barrier Option Product Specification

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April 13, 2017

Version 8.0.7970

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Chapter 1

FX Double Barrier Option

1.1 Instrument Properties

An FX double barrier option is a double barrier option with the **cross currency** as the underlying. If we denote the **primary currency** per **cross currency** exchange rate on the **maturity date** by S_T, and the agreed **strike rate** by X, provided the appropriate barrier condition is met:

1) for a **knock-out type** option, neither of the **barriers** is **touched** during the life of the option, or

2) for a **knock-in type** option, at least one of the **barriers** is **touched** during the life of the option,

and the option expires in the money, in which a **call option** expires in the money if $S_T > X$ and a **put option** expires in the money if $S_T < X$, the holder of the **call option** (**put option**) buys (sells) the **cross currency amount**, denoted by N_c, at the predetermined **strike rate** X.

Provided either of the above barrier conditions holds, the payoff of an FX double barrier option is illustrated in Table 1.1.

Option Type	Expiry Condition	Payoff (**primary currency**)
Call	$S_T \leq X$	0
	$S_T > X$	$N_c (S_T - X)$
Put	$S_T \geq X$	0
	$S_T < X$	$N_c (X - S_T)$

Table 1.1: Payoff at maturity for FX double barrier option if the underlying option is active on the maturity date

1.2 Definitions

In this section, we define terms that are specific to FX double barrier options.

barriers refers to the **lower barrier** and the **upper barrier**.

barrier direction is the direction that each of the **barriers** is considered to be **touched**.

call option gives the holder the right, but not the obligation, to buy the **cross currency** at the **strike rate** on the **maturity date** if the option is **knocked-in** (not **knocked-out**) for a **knock-in type** (**knock-out type**) option.

cross currency is the currency nominated as the underlying asset.

cross currency amount is the deal amount in **cross currency** that will be exchanged if the option is exercised.

down is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes below the **lower barrier** before or on the **maturity date**, the **lower barrier** is considered to be **touched**.

knocked-in applies to **knock-in type** options and means one of the **barriers** was **touched** and the **underlying option** became active.

knocked-out applies to **knock-out type** options and means one of the **barriers** was **touched** and the **underlying option** became inactive.

knock-in type means the **underlying option** only becomes active if one of the **barriers** is **touched** before or on the **maturity date**.

knock-out type means the **underlying option** becomes inactive if one of the **barriers** is **touched** before or on the **maturity date**.

lower barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, the **underlying option** becomes active (inactive) for **knock-in type** (**knock-out type**) options.

maturity date is the date the option expires.

primary currency is the currency that the deal is quoted in.

put option gives the holder the right, but not the obligation, to sell the **cross currency** at the **strike rate** on the **maturity date** if the option is **knocked-in** (not **knocked-out**) for a **knock-in type** (**knock-out type**) option.

settlement date is the date the **cross currency amount** is exchanged at the **strike rate** if the option is exercised.

strike rate is the agreed exchange rate between **primary currency** and **cross currency** if the option is exercised, quoted in **primary currency** per **cross currency**.

touched means the **primary currency** per **cross currency** exchange rate was above the **upper barrier** or below the **lower barrier**, before or on the **maturity date**.

underlying option is the underlying FX vanilla option that specifies the payoff of the option should the option be either **knocked-in** or not **knocked-out**.

up is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes above the **upper barrier** before or on the **maturity date**, the **upper barrier** is considered to be **touched**.

upper barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, the **underlying option** becomes active (inactive) for **knock-in type** (**knock-out type**) options.

1.3 Representations

In the Risk Engine, products are specified by *representations*. In this section, we provide the representations of FX double barrier options.

1.3.1 Default Representation

The *Default* representation consists of the mandatory trade fields in Table 1.2, the optional trade field in Table 1.3, with their restrictions in Table 1.4.

1.3.1.1 Required Curves

The following curves are required by an FX double barrier option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),

- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),

- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),

- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and

- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

Field	Description	Data Type	Symbol
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CurrencyAmount	The deal amount in *Currency*	double	N_p
CrossCurrencyAmount	The deal amount in *CrossCurrency, i.e.,* the **cross currency amount**	double	N_c
LowerBarrier	The **lower barrier** level as *Currency/CrossCurrency*	double	L
UpperBarrier	The **upper barrier** level as *Currency/CrossCurrency*	double	U
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.2: Mandatory trade fields for the Default representation of the FX Double Barrier Option

Field	Description	Data Type	Symbol	Default Value
SettlementDate	The **settlement date**	date	SD	MD

Table 1.3: Optional trade field for the Default representation of the FX Double Barrier Option

Field	Restriction
CrossCurrency	$c \neq p$
CurrencyAmount	$N_p > 0$
CrossCurrencyAmount	$N_c > 0$
LowerBarrier	$L > 0$
UpperBarrier	$U > L$
PutCall	Put, Call, P, C
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S
SettlementDate	$SD \geq MD$

Table 1.4: Trade field restrictions for the Default representation of the FX Double Barrier Option

1.3.2   Strike Representation

The *Strike* representation consists of the mandatory trade fields in Table 1.5, the optional trade field in Table 1.6, with their restrictions in Table 1.7.

1.3.2.1   Required Curves

The following curves are required by an FX double barrier option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),

- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),

- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),

- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and

- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

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Field	Description	Data Type	Symbol
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CrossCurrencyAmount	The deal amount in *CrossCurrency*, *i.e.*, the **cross currency amount**	double	N_c
Strike	The **strike rate** as *Currency/CrossCurrency*	double	X
LowerBarrier	The **lower barrier** level as *Currency/CrossCurrency*	double	L
UpperBarrier	The **upper barrier** level as *Currency/CrossCurrency*	double	U
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.5: Mandatory trade fields for the Strike representation of the FX Double Barrier Option

Field	Description	Data Type	Symbol	Default Value
SettlementDate	The **settlement date**	date	SD	MD

Table 1.6: Optional trade field for the Strike representation of the FX Double Barrier Option

Field	Restriction
CrossCurrency	$c \neq p$
CrossCurrencyAmount	$N_c > 0$
Strike	$X > 0$
LowerBarrier	$L > 0$
UpperBarrier	$U > L$
PutCall	Put, Call, P, C
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S
SettlementDate	$SD \geq MD$

Table 1.7: Trade field restrictions for the Strike representation of the FX Double Barrier Option

1.4 Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX double barrier option in Base Currency is given by the *FX double barrier option pricing function*[1],

$$\text{FXDoubleBarrier}\left(E_p, E_c, X, N_c, L, U, r_p, r_c, \sigma, T, \text{indicator}\right), \tag{1.1}$$

where

- E_p is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,

- E_c is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,

- X is the **strike rate** in units of **primary currency** per **cross currency**,

- N_c is the **cross currency amount**,

- L is the **lower barrier** in units of **primary currency** per **cross currency**,

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[1]See FX Double Barrier Option Pricing for details (p.10 of this document).

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- U is the **upper barrier** in units of **primary currency** per **cross currency**,

- r_{p} is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,

- r_{c} is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,

- σ is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,

- T is the time in years from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, and

- indicator contains the put/call, in/out and bought/sold information.

If the Valuation Date is greater than the **maturity date**, then the FX double barrier option has expired and thus has a value of zero.

1.4.1   Representation Reduction

Equation (1.1) is only defined for the Strike representation. If the trade is specified by other representations, we need to reduce it to the Strike representation.

1.4.1.1   Default Representation

For the Default representation, the **strike rate**, X, is the ratio of CurrencyAmount and CrossCurrencyAmount, given by

$$X = \frac{N_{\mathrm{p}}}{N_{\mathrm{c}}}. \tag{1.2}$$

1.5   Examples

This section provides some deal examples of FX double barrier option.

Example 1.1. An FX double barrier option in Default representation:
- Currency: AUD
- CrossCurrency: GBP
- CurrencyAmount: 100,000,000
- CrossCurrencyAmount: 60,000,000
- LowerBarrier: 1.6305
- UpperBarrier: 1.6725
- MaturityDate: 2013-11-15
- PutCall: Put
- InOut: Out
- BoughtSold: Bought

Using equation (1.2), the **strike rate** of the option is

$$X = \frac{N_{\mathrm{p}}}{N_{\mathrm{c}}} = \frac{100,000,000}{60,000,000} = 1.6667.$$

a) If on 2013-11-15, the option expires in the money with the AUD/GBP exchange rate being 1.6515, with the AUD/GBP exchange rate never went outside the range of the **lower barrier** (1.6305) and the **upper barrier** (1.6725) before 2013-11-15, the payoff of the option is

$$N_{\mathrm{c}}\left(X - S_T\right) = 60,000,000 \times (1.6667 - 1.6515) = \$910,000 \text{ AUD}.$$

b) If on 2013-11-15, the option expires out of the money with the AUD/GBP exchange rate being 1.6715, with the AUD/GBP exchange rate never went outside the range of the **lower barrier** (1.6305) and the **upper barrier** (1.6725) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (1.6667) is less than the AUD/GBP exchange rate (1.6715) on the **maturity date**.

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c) If the AUD/GBP exchange rate passed above the **upper barrier** (1.6725) before 2013-11-15, the option was **knocked-out** because the **upper barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

d) If the AUD/GBP exchange rate passed below the **lower barrier** (1.6305) before 2013-11-15, the option was **knocked-out** because the **lower barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

Example 1.2. An FX double barrier option in Strike representation:
- Currency: JPY
- CrossCurrency: USD
- CrossCurrencyAmount: 100,000,000
- Strike: 100.2
- LowerBarrier: 97.5
- UpperBarrier: 103.1
- MaturityDate: 2013-11-15
- PutCall: Call
- InOut: In
- BoughtSold: Bought

a) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 102.5, with the JPY/USD exchange rate passing below the **lower barrier** (97.5) before 2013-11-15, the payoff of the option is

$$N_c \left(S_T - X \right) = 100,000,000 \times (102.5 - 100.2) = \$230,000,000 \text{ JPY}.$$

b) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 102.5, with the JPY/USD exchange rate passing above the **upper barrier** (103.1) before 2013-11-15, the payoff of the option is

$$N_c \left(S_T - X \right) = 100,000,000 \times (102.5 - 100.2) = \$230,000,000 \text{ JPY}.$$

c) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing below the **lower barrier** (97.5) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.

d) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing above the **upper barrier** (103.1) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.

e) If the JPY/USD exchange rate never went outside the range of the **lower barrier** (97.5) and the **upper barrier** (103.1) before 2013-11-15, the option was not **knocked-in** because both of the **barriers** were not **touched** before the **maturity date**, thus the payoff of the option is 0.

Chapter 2

FX Double Barrier Option Pricing

2.1 Inputs to Function

Description	Symbol	min	max	Reasonable range
Spot rate of primary currency	E_{p}	0^+	$+\infty$	
Spot rate of cross currency	E_{c}	0^+	$+\infty$	
Strike rate as primary currency / cross currency	X	0^+	$+\infty$	
Cross currency amount	N_{c}	0^+	$+\infty$	
Lower barrier as primary currency/cross-currency	L	0^+	$< U$	
Upper barrier as primary currency/cross-currency	U	$> L$	$+\infty$	
Continuous zero rate of primary currency	r_{p}	0^+	$+\infty$	
Continuous zero rate of cross currency	r_{c}	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies	σ	0^+	$+\infty$	
Time from value date to maturity in years	T	0^+	$+\infty$	
Put or Call		–	–	"P", "C"
In or Out	indicator	–	–	"I", "O"
Bought or Sold		–	–	"B", "S"

Table 2.1: Inputs for FX Double Barrier Option pricing function

2.2 Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_{\mathrm{c}}}{E_{\mathrm{p}}}.$$

We can value an FX double barrier option by calling the *double barrier pricing function*[1] with appropriate inputs. The value of an FX double barrier option in Base Currency is

$$N_{\mathrm{c}} \times E_{\mathrm{p}} \times \mathbb{I}_{\mathrm{BS}} \times \mathrm{DoubleBarrier}\left(S, X, L, U, r_{\mathrm{p}}, r_{\mathrm{c}}, \sigma, T, \mathrm{indicator}, \mathrm{flag}\right),$$

where

$$\mathbb{I}_{\mathrm{BS}} = \begin{cases} 1, & \text{if indicator is 'B',} \\ -1, & \text{if indicator is 'S'.} \end{cases}$$

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[1]See pricing specification *Double Barrier Option* for details.

Glossary

Base Currency The currency that the risk engine is configured to return values in.

Reserve Currency The currency that all cross currency basis is benchmarked against.

Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.