FX Double Barrier at-Expiry Option
Product Specification

Vector Risk Pty Ltd

April 13, 2017

Version 8.0.7970

http://www.vectorrisk.com

Contents

List of Tables **3**

1 FX Double Barrier at-Expiry Option **4**
1.1 Instrument Properties 4
1.2 Definitions 4
1.3 Representation 5
1.4 Formula 6
1.5 Examples 6

2 FX Double Barrier at-Expiry Option Pricing **8**
2.1 Inputs to Function 8
2.2 Formula 8

Glossary **9**

List of Tables

1.1 Mandatory trade fields for the Default representation of the FX Double Barrier at-Expiry Option . . . 5
1.2 Optional trade field for the Default representation of the FX Double Barrier at-Expiry Option 5
1.3 Trade field restrictions for the Default representation of the FX Double Barrier at-Expiry Option . . . 6

2.1 Inputs for FX Double Barrier at-Expiry Option pricing function 8

Chapter 1

FX Double Barrier at-Expiry Option

1.1 Instrument Properties

An FX double barrier at-expiry option is a double barrier cash-at-expiry option with the **cross currency** as the underlying. If the appropriate barrier condition is met:

1) for a **knock-out type** option, neither of the **barriers** are **touched** during the life of the option, or

2) for a **knock-in type** option, at least one of the **barriers** are **touched** during the life of the option,

the holder of the option receives the **cash payment**, K amount of the **cash payment currency** on the **settlement date**.

1.2 Definitions

In this section, we define terms that are specific to FX double barrier at-expiry options.

barriers refers to the **lower barrier** and the **upper barrier**.

barrier direction is the direction that each of the **barriers** is considered to be **touched**.

cash payment is the amount in **cash payment currency** that the holder of the option receives on **settlement date** if option is either **knocked-in** for a **knock-in type** option or not **knocked-out** for a **knock-out type** option.

cash payment currency is the currency the **cash payment** is quoted in.

cross currency is the currency nominated as the underlying asset.

down is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes below the **lower barrier** before or on the **maturity date**, the **lower barrier** is considered to be **touched**.

knocked-in applies to **knock-in type** options and means one of the **barriers** was **touched** and there will be **cash payment** on **settlement date**.

knocked-out applies to **knock-out type** options and means one of the **barriers** was **touched** and there will be no **cash payment** on **settlement date**.

knock-in type means there is **cash payment** on **settlement date** if one of the **barriers** is **touched** before or on the **maturity date**.

knock-out type means there is no **cash payment** on **settlement date** if one of the **barriers** is **touched** before or on the **maturity date**.

lower barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, there is (is not) a **cash payment** at the **settlement date** for **knock-in type** (**knock-out type**) options.

maturity date is the date the option expires.

primary currency is the currency that the deal is quoted in.

settlement date is the date the **cash payment** is received if option is either **knocked-in** for a **knock-in type** option or not **knocked-out** for a **knock-out type** option.

touched means the **primary currency** per **cross currency** exchange rate was above the **upper barrier** or below the **lower barrier**, before or on the **maturity date**.

up is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes above the **upper barrier** before or on the **maturity date**, the **upper barrier** is considered to be **touched**.

upper barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, there is (is not) a **cash payment** on **settlement date** for **knock-in type** (**knock-out type**) options.

1.3 Representation

In the Risk Engine, products are specified by *representations*. In this section, we provide the representations of FX double barrier at-expiry options.

1.3.1 Default Representation

The *Default* representation consists of the mandatory trade fields in Table 1.1, the optional trade field in Table 1.2, with their restrictions in Table 1.3.

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
LowerBarrier	The **lower barrier** level as *Currency/CrossCurrency*	double	L
UpperBarrier	The **upper barrier** level as *Currency/CrossCurrency*	double	U
CashPaymentCurrency	The **cash payment currency**	string	kc
CashPayment	The cash payoff in *CashPaymentCurrency*, *i.e.*, the **cash payment**	double	K
MaturityDate	The **maturity date**	date	MD
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.1: Mandatory trade fields for the Default representation of the FX Double Barrier at-Expiry Option

Field	*Description*	*Data Type*	*Symbol*	*Default Value*
SettlementDate	The **settlement date**	date	SD	MD

Table 1.2: Optional trade field for the Default representation of the FX Double Barrier at-Expiry Option

1.3.1.1 Required Curves

The following curves are required by an FX double barrier at-expiry option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),
- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),
- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),
- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and
- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

Field	*Restriction*
CrossCurrency	c $\neq$ p
LowerBarrier	$L > 0$
UpperBarrier	$U > L$
CashPaymentCurrency	kc = p or kc = c
CashPayment	$K > 0$
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S
SettlementDate	SD $\geq$ MD

Table 1.3: Trade field restrictions for the Default representation of the FX Double Barrier at-Expiry Option

1.4 Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX double barrier at-expiry option in Base Currency is given by the *FX double barrier at-expiry option pricing function*[1],

$$\text{FXDoubleBarrierAtExpiry}\left(E_{\text{p}}, E_{\text{c}}, L, U, K, \mathbb{I}_{\text{kc}}, r_{\text{p}}, r_{\text{c}}, \sigma, T, \text{indicator}\right), \tag{1.1}$$

where

- E_{p} is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,
- E_{c} is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,
- L is the **lower barrier** in units of **primary currency** per **cross currency**,
- U is the **upper barrier** in units of **primary currency** per **cross currency**,
- K is the **cash payment** in **cash payment currency**,
- $\mathbb{I}_{\text{kc}}$ indicates if the **cash payment currency** is the same as the **primary currency** or as the **cross currency**,
- r_{p} is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- r_{c} is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- T is the time in years from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, and
- indicator contains the in/out and bought/sold information.

If the Valuation Date is greater than the **maturity date**, then the FX double barrier at-expiry option has expired and thus has a value of zero.

1.5 Examples

This section provides some deal examples of FX double barrier at-expiry option.

Example 1.1. An FX double barrier at-expiry option in Default representation:
- Currency: AUD
- CrossCurrency: GBP

[1] See FX Double Barrier at-Expiry Option Pricing for details (p.8 of this document).

- LowerBarrier: 1.6305
- UpperBarrier: 1.6725
- CashPaymentCurrency: AUD
- CashPayment: 100,000
- MaturityDate: 2013-11-15
- InOut: Out
- BoughtSold: Bought

a) If the AUD/GBP exchange rate never went outside the range of the **lower barrier** (1.6305) and the **upper barrier** (1.6725) before 2013-11-15, the payoff of the option is the **cash payment**, $100,000 AUD on 2013-11-15.

b) If the AUD/GBP exchange rate passed above the **upper barrier** (1.6725) before 2013-11-15, the option was **knocked-out** because the **upper barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

c) If the AUD/GBP exchange rate passed below the **lower barrier** (1.6305) before 2013-11-15, the option was **knocked-out** because the **lower barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

Example 1.2. An FX double barrier at-expiry option in Default representation:

- Currency: JPY
- CrossCurrency: USD
- LowerBarrier: 97.5
- UpperBarrier: 103.1
- CashPaymentCurrency: USD
- CashPayment: 150,000
- MaturityDate: 2013-11-15
- InOut: In
- BoughtSold: Bought

a) If the JPY/USD exchange rate passed below the **lower barrier** (97.5) before 2013-11-15, the payoff of the option is the **cash payment**, $150,000 USD on 2013-11-15.

b) If the JPY/USD exchange rate passed above the **upper barrier** (103.1) before 2013-11-15, the payoff of the option is the **cash payment**, $150,000 USD on 2013-11-15.

c) If the JPY/USD exchange rate never went outside the range of the **lower barrier** (97.5) and the **upper barrier** (103.1) before 2013-11-15, the option was not **knocked-in** because both of the **barriers** were not **touched** before the **maturity date**, thus the payoff of the option is 0.

Chapter 2

FX Double Barrier at-Expiry Option Pricing

2.1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Spot rate of primary currency	E_{p}	0^+	$+\infty$	
Spot rate of cross currency	E_{c}	0^+	$+\infty$	
Lower barrier as primary currency/cross-currency	L	0^+	$< U$	
Upper barrier as primary currency/cross-currency	U	$> L$	$+\infty$	
Indicator for cash payment currency	$\mathbb{I}_{\mathrm{kc}}$			"Currency", "CrossCurrency"
Cash amount of payoff in cash payment currency	K	0^+	$+\infty$	
Continuous zero rate of primary currency	r_{p}	0^+	$+\infty$	
Continuous zero rate of cross currency	r_{c}	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies	σ	0^+	$+\infty$	
Time from value date to maturity in years	T	0^+	$+\infty$	
In or Out		–	–	"I", "O"
Bought or Sold		–	–	"B", "S"

Table 2.1: Inputs for FX Double Barrier at-Expiry Option pricing function

2.2 Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_{\mathrm{c}}}{E_{\mathrm{p}}}.$$

We can value an FX double barrier at-expiry option by calling the *double barrier cash at-expiry pricing function*[1] or the *double barrier asset at-expiry pricing function*[2] with appropriate inputs. The value of an FX double barrier at-expiry option in Base Currency is

$$\begin{cases} E_{\mathrm{p}} \times \mathbb{I}_{\mathrm{BS}} \times \mathrm{DoubleBarrierCashAtExpiry}\left(S, L, U, K, r_{\mathrm{p}}, r_{\mathrm{c}}, \sigma, T, \mathrm{indicator}\right), & \text{if } \mathbb{I}_{\mathrm{kc}} = \mathrm{Currency}, \\ E_{\mathrm{p}} \times \mathbb{I}_{\mathrm{BS}} \times K \times \mathrm{DoubleBarrierAssetAtExpiry}\left(S, L, U, r_{\mathrm{p}}, r_{\mathrm{c}}, \sigma, T, \mathrm{indicator}\right), & \text{if } \mathbb{I}_{\mathrm{kc}} = \mathrm{CrossCurrency}, \end{cases}$$

where

$$\mathbb{I}_{\mathrm{BS}} = \begin{cases} 1, & \text{if indicator is 'B'}, \\ -1, & \text{if indicator is 'S'}. \end{cases}$$

[1] See pricing specification *Double Barrier Cash-at-Expiry Option* for details.
[2] See pricing specification *Double Barrier Asset-at-Expiry Option* for details.

Glossary

Base Currency The currency that the risk engine is configured to return values in.

Reserve Currency The currency that all cross currency basis is benchmarked against.

Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.