FX Double Barrier at-Touch Option
Product Specification

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April 13, 2017

Version 8.0.7970

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Chapter 1

FX Double Barrier at-Touch Option

1.1 Instrument Properties

An FX double barrier at-touch option is a double barrier cash-at-touch option with the **cross currency** as the underlying. The three possible outcomes of an FX double barrier at-touch option are:

1) neither of the **barriers** is **touched** during the life of the option, the option expires without payout,

2) the **lower barrier** was **touched** before the **upper barrier** was **touched** during the life of the option, there was a payout of **lower cash payment**, K_L in **cash payment currency** as soon as the **lower barrier** was **touched**, and

3) the **upper barrier** was **touched** before the **lower barrier** was **touched** during the life of the option, there was a payout of **upper cash payment**, K_U in **cash payment currency** as soon as the **upper barrier** was **touched**.

1.2 Definitions

In this section, we define terms that are specific to FX double barrier at-touch options.

barriers refers to the **lower barrier** and the **upper barrier**.

barrier direction is the direction that each of the **barriers** is considered to be **touched**.

cash payment currency is the currency the **lower cash payment** and the **upper cash payment** are quoted in.

cross currency is the currency nominated as the underlying asset.

down is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes below the **lower barrier** before or on the **maturity date**, the **lower barrier** is considered to be **touched**.

lower barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, there is an immediate payout of the **lower cash payment**.

lower cash payment is the amount in **cash payment currency** that the holder receives immediately when the **lower barrier** is **touched**.

maturity date is the date the option expires.

primary currency is the currency that the deal is quoted in.

touched means the **primary currency** per **cross currency** exchange rate was above the **upper barrier** or below the **lower barrier**, before or on the **maturity date**.

up is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes above the **upper barrier** before or on the **maturity date**, the **upper barrier** is considered to be **touched**.

upper barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, there is an immediate payout of the **upper cash payment**.

upper cash payment is the amount in **cash payment currency** that the holder receives immediately when the **upper barrier** is **touched**.

1.3 Representation

In the Risk Engine, products are specified by *representations*. In this section, we provide the representation of FX double barrier at-touch options.

1.3.1 Default Representation

The *Default* representation consists of the mandatory trade fields in Table 1.1, the optional trade field in Table 1.2, with their restrictions in Table 1.3.

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
LowerBarrier	The **lower barrier** level as *Currency*/*CrossCurrency*	double	L
UpperBarrier	The **upper barrier** level as *Currency*/*CrossCurrency*	double	U
CashPaymentCurrency	The **cash payment currency**	string	kc
LowerCashPayment	The cash payoff in *CashPaymentCurrency* when the **lower barrier** is **touched**, *i.e.*, the **lower cash payment**	double	K_L
MaturityDate	The **maturity date**	date	MD
BoughtSold	Bought or sold the option	string	BS

Table 1.1: Mandatory trade fields for the Default representation of the FX Double Barrier at-Touch Option

Field	*Description*	*Data Type*	*Symbol*	*Default Value*
UpperCashPayment	The cash payoff in *CashPaymentCurrency* when the **lower barrier** is **touched**, *i.e.*, the **upper cash payment**	double	K_U	K_L

Table 1.2: Optional trade field for the Default representation of the FX Double Barrier at-Touch Option

Field	*Restriction*
CrossCurrency	$c \neq p$
LowerBarrier	$L > 0$
UpperBarrier	$U > L$
CashPaymentCurrency	kc = p or kc = c
LowerCashPayment	$K_L > 0$
BoughtSold	Bought, Sold, B, S
UpperCashPayment	$K_U > 0$

Table 1.3: Trade field restrictions for the Default representation of the FX Double Barrier at-Touch Option

1.3.1.1 Required Curves

The following curves are required by an FX double barrier at-touch option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),
- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),
- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),
- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and
- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

1.4 Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX double barrier at-touch option in Base Currency is given by the *FX double barrier at-touch option pricing function*[1],

$$\text{FXDoubleBarrierAtTouch}\left(E_{\text{p}}, E_{\text{c}}, L, U, K_L, K_U, \mathbb{I}_{\text{kc}}, r_{\text{p}}, r_{\text{c}}, \sigma, T, \text{indicator}\right), \tag{1.1}$$

where

- E_{p} is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,
- E_{c} is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,
- L is the **lower barrier** in units of **primary currency** per **cross currency**,
- U is the **upper barrier** in units of **primary currency** per **cross currency**,
- K_L is **lower cash payment** in **cash payment currency**, when the **lower barrier** is **touched**,
- K_U is **upper cash payment** in **cash payment currency**, when the **upper barrier** is **touched**,
- $\mathbb{I}_{\text{kc}}$ indicates if the **cash payment currency** is the same as the **primary currency** or as the **cross currency**,
- r_{p} is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- r_{c} is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- T is the time in years from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, and
- indicator contains bought/sold information.

If the Valuation Date is greater than the **maturity date**, then the FX double barrier at-touch option has expired and thus has a value of zero.

1.5 Examples

This section provides some deal examples of FX double barrier at-touch option.

Example 1.1. An FX double barrier at-touch option in Default representation:

- Currency: AUD
- CrossCurrency: GBP
- LowerBarrier: 1.6305
- UpperBarrier: 1.6725
- CashPaymentCurrency: AUD
- LowerCashPayment: 100,000
- UpperCashPayment: 150,000
- MaturityDate: 2013-11-15
- BoughtSold: Bought

a) If the AUD/GBP exchange rate passed above the **upper barrier** (1.6725) on 2013-10-01 before the **lower barrier** is **touched**, the payoff of the option is the **upper cash payment**, \$150,000 AUD on 2013-10-01.

b) If the AUD/GBP exchange rate passed below the **lower barrier** (1.6305) on 2013-10-01 before the **upper barrier** is **touched**, the payoff of the option is the **lower cash payment**, \$100,000 AUD on 2013-10-01.

c) If the AUD/GBP exchange rate never went outside the range of the **lower barrier** (1.6305) and the **upper barrier** (1.6725) before 2013-11-15, the payoff of the option is 0 as neither of the **barriers** was **touched** before the **maturity date**.

[1] See FX Double Barrier at-Touch Option Pricing for details (p.7 of this document).

Chapter 2

FX Double Barrier at-Touch Option Pricing

2.1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Spot rate of primary currency	E_{p}	0^+	$+\infty$	
Spot rate of cross currency	E_{c}	0^+	$+\infty$	
Lower barrier as primary currency/cross-currency	L	0^+	$< U$	
Upper barrier as primary currency/cross-currency	U	$> L$	$+\infty$	
Indicator for cash payment currency	$\mathbb{I}_{\mathrm{kc}}$			"Currency", "CrossCurrency"
Cash amount of payoff in cash payment currency if the lower barrier is hit	K_L	0^+	$+\infty$	
Cash amount of payoff in cash payment currency if the upper barrier is hit	K_U	0^+	$+\infty$	
Continuous zero rate of primary currency	r_{p}	0^+	$+\infty$	
Continuous zero rate of cross currency	r_{c}	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies	σ	0^+	$+\infty$	
Time from value date to maturity in years	T	0^+	$+\infty$	
Bought or Sold		–	–	"B", "S"

Table 2.1: Inputs for FX Double Barrier at-Touch Option pricing function

2.2 Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_{\mathrm{c}}}{E_{\mathrm{p}}}.$$

We can value an FX double barrier at-touch option by calling the *double barrier cash at-touch pricing function*[1] with appropriate inputs. The value of an FX double barrier at-touch option in Base Currency is

$$\begin{cases} E_{\mathrm{p}} \times \mathbb{I}_{\mathrm{BS}} \times \mathrm{DoubleBarrierCashAtTouch}\left(S, L, U, K_L, K_U, r_{\mathrm{p}}, r_{\mathrm{c}}, \sigma, T, \mathrm{indicator}\right), & \text{if } \mathbb{I}_{\mathrm{kc}} = \mathrm{Currency}, \\ E_{\mathrm{c}} \times \mathbb{I}_{\mathrm{BS}} \times \mathrm{DoubleBarrierCashAtTouch}\left(S^{-1}, U^{-1}, L^{-1}, K_U, K_L, r_{\mathrm{c}}, r_{\mathrm{p}}, \sigma, T, \mathrm{indicator}\right), & \text{if } \mathbb{I}_{\mathrm{kc}} = \mathrm{CrossCurrency}, \end{cases}$$

[1] See pricing specification *Double Barrier Cash-at-Touch Option* for details.

where

$$\mathbb{I}_{\text{BS}} = \begin{cases} 1, & \text{if indicator is 'B',} \\ -1, & \text{if indicator is 'S'.} \end{cases}$$

Glossary

Base Currency The currency that the risk engine is configured to return values in.

Reserve Currency The currency that all cross currency basis is benchmarked against.

Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.