FX Double Window Barrier Option
Product Specification

Vector Risk Pty Ltd

April 13, 2017

Version 8.0.7970

http://www.vectorrisk.com

Contents

List of Tables

Chapter 1

FX Double Window Barrier Option

1.1 Instrument Properties

An FX double window barrier option is a double window barrier option with the **cross currency** as the underlying. If we denote the **primary currency** per **cross currency** exchange rate on the **maturity date** by S_T, and the agreed **strike rate** by X, provided the appropriate barrier condition is met:

1) for a **knock-out type** option, neither of the **barriers** is **touched** between the **barrier start date** and the **barrier end date**, or

2) for a **knock-in type** option, at least one of the **barriers** is **touched** between the **barrier start date** and the **barrier end date**,

and the option expires in the money, in which a **call option** expires in the money if $S_T > X$ and a **put option** expires in the money if $S_T < X$, the holder of the **call option** (**put option**) buys (sells) the **cross currency amount**, denoted by N_c, at the predetermined **strike rate** X.

Provided either of the above barrier conditions holds, the payoff of an FX double window barrier option is illustrated in Table 1.1.

Option Type	Expiry Condition	Payoff (**primary currency**)
Call	$S_T \leq X$	0
	$S_T > X$	$N_c\,(S_T - X)$
Put	$S_T \geq X$	0
	$S_T < X$	$N_c\,(X - S_T)$

Table 1.1: Payoff at maturity for FX double window barrier option if the underlying potion is active on the maturity date

1.2 Definitions

In this section, we define terms that are specific to FX double window barrier options.

barriers refers to the **lower barrier** and the **upper barrier**.

barrier direction is the direction that each of the **barriers** is considered to be **touched**.

barrier end date is the date the **barriers** become inactive.

barrier start date is the date the **barriers** become active.

call option gives the holder the right, but not the obligation, to buy the **cross currency** at the **strike rate** on the **maturity date** if the option is **knocked-in** (not **knocked-out**) for a **knock-in type** (**knock-out type**) option.

cross currency is the currency nominated as the underlying asset.

cross currency amount is the deal amount in **cross currency** that will be exchanged if the option is exercised.

down is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes below the **lower barrier** between the **barrier start date** and the **barrier end date**, the **lower barrier** is considered to be **touched**.

knocked-in applies to **knock-in type** options and means one of the **barriers** was **touched** and the **underlying option** became active.

knocked-out applies to **knock-out type** options and means one of the **barriers** was **touched** and the **underlying option** became inactive.

knock-in type means the **underlying option** only becomes active if one of the **barriers** is **touched** between the **barrier start date** and the **barrier end date**.

knock-out type means the **underlying option** becomes inactive if one of the **barriers** is **touched** between the **barrier start date** and the **barrier end date**.

lower barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** between the **barrier start date** and the **barrier end date**, the **underlying option** becomes active (inactive) for **knock-in type** (**knock-out type**) options.

maturity date is the date the option expires.

primary currency is the currency that the deal is quoted in.

put option gives the holder the right, but not the obligation, to sell the **cross currency** at the **strike rate** on the **maturity date** if the option is **knocked-in** (not **knocked-out**) for a **knock-in type** (**knock-out type**) option.

strike rate is the agreed exchange rate between **primary currency** and **cross currency** if the option is exercised, quoted in **primary currency** per **cross currency**.

touched means the **primary currency** per **cross currency** exchange rate was above the **upper barrier** or below the **lower barrier**, between the **barrier start date** and the **barrier end date**.

underlying option is the underlying FX vanilla option that specifies the payoff of the option should the option be either **knocked-in** or not **knocked-out**.

up is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes above the **upper barrier** between the **barrier start date** and the **barrier end date**, the **upper barrier** is considered to be **touched**.

upper barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** between the **barrier start date** and the **barrier end date**, the **underlying option** becomes active (inactive) for **knock-in type** (**knock-out type**) options.

1.3 Representations

In the Risk Engine, products are specified by *representations*. In this section, we provide the representations of FX double window barrier options.

1.3.1 Default Representation

The *Default* representation consists of the mandatory trade fields in Table 1.2, with their restrictions in Table 1.3.

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CurrencyAmount	The deal amount in *Currency*	double	N_{p}
CrossCurrencyAmount	The deal amount in *CrossCurrency*, *i.e.*, the **cross currency amount**	double	N_{c}
LowerBarrier	The **lower barrier** level as *Currency*/*CrossCurrency*	double	L
UpperBarrier	The **upper barrier** level as *Currency*/*CrossCurrency*	double	U
BarrierStart	The **barrier start date**	date	BSD
BarrierEnd	The **barrier end date**	date	BED
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.2: Mandatory trade fields for the Default representation of the FX Double Window Barrier Option

Field	*Restriction*
CrossCurrency	$\mathrm{c} \neq \mathrm{p}$
CurrencyAmount	$N_{\mathrm{p}} > 0$
CrossCurrencyAmount	$N_{\mathrm{c}} > 0$
LowerBarrier	$L > 0$
UpperBarrier	$U > L$
BarrierEnd	BED > BSD
MaturityDate	MD > BED
PutCall	Put, Call, P, C
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S

Table 1.3: Trade field restrictions for the Default representation of the FX Double Window Barrier Option

1.3.1.1 Required Curves

The following curves are required by an FX double window barrier option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),
- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),
- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),
- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and
- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

1.3.2 Strike Representation

The *Strike* representation consists of the mandatory trade fields in Table 1.4, with their restrictions in Table 1.5.

1.3.2.1 Required Curves

The following curves are required by an FX double window barrier option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),
- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),
- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CrossCurrencyAmount	The deal amount in *CrossCurrency*, *i.e.*, the **cross currency amount**	double	N_c
Strike	The **strike rate** as *Currency*/*CrossCurrency*	double	X
LowerBarrier	The **lower barrier** level as *Currency*/*CrossCurrency*	double	L
UpperBarrier	The **upper barrier** level as *Currency*/*CrossCurrency*	double	U
BarrierStart	The **barrier start date**	date	BSD
BarrierEnd	The **barrier end date**	date	BED
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.4: Mandatory trade fields for the Strike representation of the FX Double Window Barrier Option

Field	*Restriction*
CrossCurrency	$c \neq p$
CrossCurrencyAmount	$N_c > 0$
Strike	$X > 0$
LowerBarrier	$L > 0$
UpperBarrier	$U > L$
BarrierEnd	BED > BSD
MaturityDate	MD > BED
PutCall	Put, Call, P, C
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S

Table 1.5: Trade field restrictions for the Strike representation of the FX Double Window Barrier Option

- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and
- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

1.4 Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX double window barrier option in Base Currency is given by the *FX double window barrier option pricing function*[1],

$$\text{FXDoubleWindowBarrier}\left(E_p, E_c, X, N_c, L, U, r_{p,1}, r_{c,1}, \sigma_1, t_1, r_{p,2}, r_{c,2}, \sigma_2, t_2, r_{p,3}, r_{c,3}, \sigma_3, T_3, \text{indicator}\right), \tag{1.1}$$

where

- E_p is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,
- E_c is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,
- X is the **strike rate** in units of **primary currency** per **cross currency**,
- N_c is the **cross currency amount**,
- L is the **lower barrier** in units of **primary currency** per **cross currency**,

[1] See FX Double Window Barrier Option Pricing for details (p.11 of this document).

- U is the **upper barrier** in units of **primary currency** per **cross currency**,
- $r_{\mathrm{p},1}$ is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{\mathrm{c},1}$ is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ_1 is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- t_1 is the time in years from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention,
- $r_{\mathrm{p},2}$ is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{\mathrm{c},2}$ is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ_2 is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- t_2 is the time in years from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention,
- $r_{\mathrm{p},3}$ is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{\mathrm{c},3}$ is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ_3 is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- T_3 is the time in years from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, and
- indicator contains the put/call, in/out and bought/sold information.

If the Valuation Date is greater than the **maturity date**, then the FX double window barrier option has expired and thus has a value of zero.

1.4.1 Representation Reduction

Equation (1.1) is only defined for the Strike representation. If of the trade is specified by other representations, we need to reduce it to the Strike representation.

1.4.1.1 Default Representation

For the Default representation, the **strike rate**, X, is the ratio of CurrencyAmount and CrossCurrencyAmount, given by

$$X = \frac{N_{\mathrm{p}}}{N_{\mathrm{c}}}. \tag{1.2}$$

1.5 Examples

This section provides some deal examples of FX double window barrier option.

Example 1.1. An FX double window barrier option in Default representation:

- Currency: AUD
- CrossCurrency: GBP
- CurrencyAmount: 100,000,000
- CrossCurrencyAmount: 60,000,000
- LowerBarrier: 1.6305
- UpperBarrier: 1.6725
- BarrierStartDate: 2013-10-15
- BarrierEndDate: 2013-10-31
- MaturityDate: 2013-11-15
- PutCall: Put
- InOut: Out
- BoughtSold: Bought

Using equation (1.2), the **strike rate** of the option is

$$X = \frac{N_{\mathrm{p}}}{N_{\mathrm{c}}} = \frac{100,000,000}{60,000,000} = 1.6667.$$

a) If on 2013-11-15, the option expires in the money with the AUD/GBP exchange rate being 1.6515, with the AUD/GBP exchange rate never went outside the range of the **lower barrier** (1.6305) and the **upper barrier** (1.6725) between 2013-10-15 and 2013-10-31, the payoff of the option is

$$N_{\mathrm{c}}\left(X - S_T\right) = 60,000,000 \times (1.6667 - 1.6515) = \$910,000 \text{ AUD}.$$

b) If on 2013-11-15, the option expires out of the money with the AUD/GBP exchange rate being 1.6715, with the AUD/GBP exchange rate never went outside the range of the **lower barrier** (1.6305) and the **upper barrier** (1.6725) between 2013-10-15 and 2013-10-31, the payoff of the option is 0 as the **strike rate** (1.6667) is less than the AUD/GBP exchange rate (1.6715) on the **maturity date**.

c) If the AUD/GBP exchange rate passed above the **upper barrier** (1.6725) between 2013-10-15 and 2013-10-31, the option was **knocked-out** because the **upper barrier** was **touched** between the **barrier start date** and the **barrier end date**, thus the payoff of the option is 0.

d) If the AUD/GBP exchange rate passed below the **lower barrier** (1.6305) between 2013-10-15 and 2013-10-31, the option was **knocked-out** because the **lower barrier** was **touched** between the **barrier start date** and the **barrier end date**, thus the payoff of the option is 0.

Example 1.2. An FX double window barrier option in Strike representation:

- Currency: JPY
- CrossCurrency: USD
- CrossCurrencyAmount: 100,000,000
- Strike: 100.2
- LowerBarrier: 97.5
- UpperBarrier: 103.1
- BarrierStartDate: 2013-10-15
- BarrierEndDate: 2013-10-31
- MaturityDate: 2013-11-15
- PutCall: Call
- InOut: In
- BoughtSold: Bought

a) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 102.5, with the JPY/USD exchange rate passing below the **lower barrier** (97.5) between 2013-10-15 and 2013-10-31, the payoff of the option is

$$N_{\mathrm{c}}\left(S_T - X\right) = 100,000,000 \times (102.5 - 100.2) = \$230,000,000 \text{ JPY}.$$

b) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 102.5, with the JPY/USD exchange rate passing above the **upper barrier** (103.1) between 2013-10-15 and 2013-10-31, the payoff of the option is

$$N_{\mathrm{c}}\left(S_T - X\right) = 100,000,000 \times (102.5 - 100.2) = \$230,000,000 \text{ JPY}.$$

c) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing below the **lower barrier** (97.5) between 2013-10-15 and 2013-10-31, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.

d) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing above the **upper barrier** (103.1) between 2013-10-15 and 2013-10-31, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.

e) If the JPY/USD exchange rate never went outside the range of the **lower barrier** (97.5) and the **upper barrier** (103.1) between 2013-10-15 and 2013-10-31, the option was not **knocked-in** because both of the **barriers** were not **touched** between the **barrier start date** and the **barrier end date**, thus the payoff of the option is 0.

Chapter 2

FX Double Window Barrier Option Pricing

2.1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Spot rate of primary currency	E_{p}	0^+	$+\infty$	
Spot rate of cross currency	E_{c}	0^+	$+\infty$	
Strike rate as primary currency/cross currency	X	0^+	$+\infty$	
Cross currency amount	N_{c}	0^+	$+\infty$	
Lower barrier as primary currency/cross-currency	L	0^+	$< U$	
Upper barrier as primary currency/cross-currency	U	$> L$	$+\infty$	
Continuous zero rate of primary currency till t_1	$r_{\mathrm{p},1}$	0^+	$+\infty$	
Continuous zero rate of cross currency till t_1	$r_{\mathrm{c},1}$	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies till t_1	σ_1	0^+	$+\infty$	
Time from value date to barrier start date in years	t_1	0^+	$< t_2$	
Continuous zero rate of primary currency till t_2	$r_{\mathrm{p},2}$	0^+	$+\infty$	
Continuous zero rate of cross currency till t_2	$r_{\mathrm{c},2}$	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies till t_2	σ_2	0^+	$+\infty$	
Time from value date to barrier end date in years	t_2	$> t_1$	$< T_3$	
Continuous zero rate of primary currency till T_3	$r_{\mathrm{p},3}$	0^+	$+\infty$	
Continuous zero rate of cross currency till T_3	$r_{\mathrm{c},3}$	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies till T_3	σ_3	0^+	$+\infty$	
Time from value date to maturity in years	T_3	$> t_2$	$+\infty$	
Put or Call		–	–	"P", "C"
In or Out	indicator	–	–	"I", "O"
Bought or Sold		–	–	"B", "S"

Table 2.1: Inputs for FX Double Window Barrier Option pricing function

2.2 Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_{\mathrm{c}}}{E_{\mathrm{p}}}.$$

We can value an FX double window barrier option by calling the *double window barrier pricing function*[1] with appropriate inputs. The value of an FX double window barrier option in Base Currency is

$$N_{\mathrm{c}} \times E_{\mathrm{p}} \times \mathbb{I}_{\mathrm{BS}} \times \mathrm{DoubleWindowBarrier}\left(S, X, L, U, r_{\mathrm{p},1}, r_{\mathrm{c},1}, \sigma_1, t_1, r_{\mathrm{p},2}, r_{\mathrm{c},2}, \sigma_2, t_2, r_{\mathrm{p},3}, r_{\mathrm{c},3}, \sigma_3, T_3, \mathrm{indicator}\right),$$

where

$$\mathbb{I}_{\mathrm{BS}} = \begin{cases} 1, & \text{if indicator is 'B',} \\ -1, & \text{if indicator is 'S'.} \end{cases}$$

[1] See pricing specification *Double Window Barrier Option* for details.

Glossary

Base Currency The currency that the risk engine is configured to return values in.

Reserve Currency The currency that all cross currency basis is benchmarked against.

Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.