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FX Knock-In Knock-Out Barrier Option Product Specification

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Contents

List of Tables

1	FX Knock-In Knock-Out Barrier Option	4
-	1.1 Instrument Properties	4
	1.2 Definitions	5
	1.3 Representations	5
	1.4 Trade Field Restrictions	7
	1.5 Formula	8
	1.6 Examples	9
2	FX Knock-In Knock-Out Barrier Option Pricing	12
	2.1 Inputs to Function	12
	2.2 Formula	12
G	Hossary	13

3

List of Tables

1.1	Payoff at maturity for FX knock-in knock-out barrier option if the underlying potion is active on the	
	maturity date	5
1.2	Mandatory trade fields for the Default representation of the FX Knock-In Knock-Out Barrier Option .	6
1.3	Trade field restrictions for the Default representation of the FX Knock-In Knock-Out Barrier Option .	6
1.4	Mandatory trade fields for the Strike representation of the FX Knock-In Knock-Out Barrier Option .	7
1.5	Trade field restrictions for the Strike representation of the FX Knock-In Knock-Out Barrier Option	7
1.6	Restriction on the relationship between the barriers for FX knock-in knock-out barrier option \ldots .	7
0.1		10
2.1	Inputs for FA Knock-In Knock-Out Barrier Option pricing function	12

Chapter 1

FX Knock-In Knock-Out Barrier Option

1.1 Instrument Properties

An FX knock-in knock-out barrier option is a knock-in knock-out barrier option with the **cross currency** as the underlying. If we denote the **primary currency** per **cross currency** exchange rate on the **maturity date** by S_T , and the agreed **strike rate** by X, provided that only the **in barrier** was **touched** during the life of the option, but not the **out barrier**, and the option expires in the money, in which a **call option** expires in the money if $S_T > X$ and a **put option** expires in the money if $S_T < X$, the holder of the **call option** (**put option**) buys (sells) the **cross currency amount**, denoted by N_c , at the predetermined **strike rate** X.

It is important to note that for an FX knock-in knock-out barrier option,

- 1) if the **out barrier** is **touched** during the life of the option, the option is **knocked-out** and ceases to exist regardless whether the **in barrier** was **touched** during the life of the option, and
- 2) if the in barrier is not touched during the life of the option, the option is not knocked-in and has no payout.

That is, the holder of the option only has the right to exercise the option on the **maturity date** if the option is **knocked-in** but not **knocked-out**.

The four versions of FX knock-in knock-out barrier option are:

- 1) both the **in barrier** and the **out barrier** are **up** barriers and by definition, the **in barrier** is below the **out barrier**,
- 2) the **in barrier** is an **up** barrier and the **out barrier** is a **down** barrier and by definition, the **in barrier** is above the **out barrier**,
- 3) the **in barrier** is a **down** barrier and the **out barrier** is an **up** barrier and by definition, the **in barrier** is below the **out barrier**, and
- 4) both the **in barrier** and the **out barrier** are **down** barriers and by definition, the **in barrier** is above the **out barrier**.

For cases 1) and 4), if the relationship between the **in barrier** and the **out barrier** is violated, the option will always get **knocked-out** before **knocked-in**. For cases 2) and 3), if the relationship between the **in barrier** and the **out barrier** is violated, the option becomes an FX single barrier option with the **out barrier**, since if the option is not **knocked-in**, then it is already **knocked-out**.

Provided the **in barrier** is **touched** and the **out barrier** is not **touched** during the life of the option, the payoff of an FX knock-in knock-out barrier option is illustrated in Table 1.1.

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Option Type	Expiry Condition	Payoff (primary currency)
Call	$S_T \le X$ $S_T > X$	$egin{array}{c} 0 \ N_{ m c} \left(S_T - X ight) \end{array}$
Put	$S_T \ge X$ $S_T < X$	$\frac{0}{N_{\rm c}\left(X-S_T\right)}$

Table 1.1: Payoff at maturity for FX knock-in knock-out barrier option if the underlying potion is active on the maturity date

1.2 Definitions

In this section, we define terms that are specific to FX knock-in knock-out barrier options.

barriers refers to the in barrier and the out barrier.

barrier direction is the direction that each of the barriers is considered to be touched.

call option gives the holder the right, but not the obligation, to buy the cross currency at the strike rate on the maturity date if the option is knocked-in but not knocked-out.

cross currency is the currency nominated as the underlying asset.

cross currency amount is the deal amount in cross currency that will be exchanged if the option is exercised.

down is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes below the barrier on or before the **maturity date**, the barrier is considered to be **touched**. This direction can apply to the **in barrier** or the **out barrier**.

knocked-in means the in barrier was touched.

knocked-out means the out barrier was touched.

- in barrier is the the primary currency per cross currency exchange rate level such that, if it is touched before or on the maturity date, the option is knocked-in.
- maturity date is the date the option expires.
- **out barrier** is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, the option is **knocked-out**.
- primary currency is the currency that the deal is quoted in.
- put option gives the holder the right, but not the obligation, to sell the cross currency at the strike rate on the maturity date if the option is knocked-in but not knocked-out.
- strike rate is the agreed exchange rate between primary currency and cross currency if the option is exercised, quoted in primary currency per cross currency.
- touched means the primary currency per cross currency exchange rate was above (below) either the in barrier or the out barrier in the case where the barrier direction is up (down).
- **up** is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes above the barrier before or on the **maturity date**, the barrier is considered to be **touched**. This direction can apply to the **in barrier** or the **out barrier**.

1.3 Representations

In the Risk Engine, products are specified by *representations*. In this section, we provide the representations of FX knock-in knock-out barrier options.



Field	Description	Data Type	Symbol
Currency	The primary currency	string	р
CrossCurrency	The cross currency	string	с
CurrencyAmount	The deal amount in <i>Currency</i>	double	$N_{\rm p}$
CrossCurrencyAmount	The deal amount in CrossCurrency, i.e., the cross	double	$\dot{N_{ m c}}$
	currency amount		
InBarrier	The in barrier level as <i>Currency</i> / <i>CrossCurrency</i>	double	Ι
OutBarrier	The out barrier level as <i>Currency</i> / <i>CrossCurrency</i>	double	0
MaturityDate	The maturity date	date	MD
PutCall	Put option or call option on CrossCurrency	string	\mathbf{PC}
InUpDown	Direction of the in barrier	string	iud
OutUpDown	Direction of the out barrier	string	oud
BoughtSold	Bought or sold the option	string	BS

Table 1.2: Mandatory trade fields for the Default representation of the FX Knock-In Knock-Out Barrier Option

Field	Restriction
CrossCurrency	$c \neq p$
CurrencyAmount	$N_{\rm p} > 0$
CrossCurrencyAmount	$\dot{N_{\rm c}} > 0$
InBarrier	I > 0
OutBarrier	O > 0
PutCall	Put, Call, P, C
InUpDown	Up, Down, U, D
OutUpDown	Up, Down, U, D
BoughtSold	Bought, Sold, B, S

Table 1.3: Trade field restrictions for the Default representation of the FX Knock-In Knock-Out Barrier Option

1.3.1 Default Representation

The Default representation consists of the mandatory trade fields in Table 1.2, with their restrictions in Table 1.3.

1.3.1.1 Required Curves

The following curves are required by an FX knock-in knock-out barrier option:

- Currency FX spot curve: FX Spot Curve (FX.PRICE.Currency.BaseCurrency),
- CrossCurrency FX spot curve: FX Spot Curve (FX.PRICE.CrossCurrency.BaseCurrency),
- Currency discounting curve: FX Zero Curve (FX.ZERO.Currency.ReserveCurrency),
- CrossCurrency discounting curve: FX Zero Curve (FX.ZERO.CrossCurrency.ReserveCurrency), and
- Currency, CrossCurrency volatility grid: FX Volatility Grid (FX.GRID.CrossCurrency.Currency).

1.3.2 Strike Representation

The *Strike* representation consists of the mandatory trade fields in Table 1.4, with their restrictions in Table 1.5.

1.3.2.1 Required Curves

The following curves are required by an FX knock-in knock-out barrier option:

- Currency FX spot curve: FX Spot Curve (FX.PRICE.Currency.BaseCurrency),
- CrossCurrency FX spot curve: FX Spot Curve (FX.PRICE.CrossCurrency.BaseCurrency),

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Field	Description	Data Type	Symbol
Currency	The primary currency	string	р
CrossCurrency	The cross currency	string	с
CrossCurrencyAmount	The deal amount in CrossCurrency, i.e., the cross	double	$N_{ m c}$
	currency amount		
Strike	The strike rate as <i>Currency</i> / <i>CrossCurrency</i>	double	X
InBarrier	The in barrier level as <i>Currency/CrossCurrency</i>	double	Ι
OutBarrier	The out barrier level as <i>Currency</i> / <i>CrossCurrency</i>	double	0
MaturityDate	The maturity date	date	MD
PutCall	Put option or call option on CrossCurrency	string	\mathbf{PC}
InUpDown	Direction of the in barrier	string	iud
OutUpDown	Direction of the out barrier	string	oud
BoughtSold	Bought or sold the option	string	BS

Table 1.4: Mandatory trade fields for the Strike representation of the FX Knock-In Knock-Out Barrier Option

Field	Restriction
CrossCurrency	$c \neq p$
CrossCurrencyAmount	$N_{\rm c} > 0$
Strike	X > 0
InBarrier	I > 0
OutBarrier	O > 0
PutCall	Put, Call, P, C
InUpDown	Up, Down, U, D
OutUpDown	Up, Down, U, D
BoughtSold	Bought, Sold, B, S

Table 1.5: Trade field restrictions for the Strike representation of the FX Knock-In Knock-Out Barrier Option

- Currency discounting curve: FX Zero Curve (FX.ZERO.Currency.ReserveCurrency),
- CrossCurrency discounting curve: FX Zero Curve (FX.ZERO.CrossCurrency.ReserveCurrency), and
- Currency, CrossCurrency volatility grid: FX Volatility Grid (FX.GRID.CrossCurrency.Currency).

1.4 Trade Field Restrictions

Furthermore, the valid relationship between the trade fields 'InBarrier' and 'OutBarrier' depends on the value of the trade field 'OutUpDown'.

Section 1.1 stated that for the cases of the **out barrier** is an **up** barrier, the **in barrier** has to be below the **out barrier**, and for the cases of the **out barrier** is an **down** barrier, the **in barrier** has to be above the **out barrier**, as summarised in Table 1.6.

OutUpDown	Restriction
Up Down	I < O $I > O$

Table 1.6: Restriction on the relationship between the barriers for FX knock-in knock-out barrier option



1.5 Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX knock-in knock-out barrier option in Base Currency is given by the FX knock-in knock-out barrier option pricing function¹,

FXKnockInKnockOutBarrier $(E_{\rm p}, E_{\rm c}, X, N_{\rm c}, I, O, r_{\rm p}, r_{\rm c}, \sigma, T, \text{inIndicator}, \text{outIndicator}),$ (1.1)

where

- $E_{\rm p}$ is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,
- E_c is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,
- X is the strike rate in units of primary currency per cross currency,
- $N_{\rm c}$ is the cross currency amount,
- *I* is the **in barrier** in units of **primary currency** per **cross currency**,
- O is the **out barrier** in units of **primary currency** per **cross currency**,
- $r_{\rm p}$ is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{\rm c}$ is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- T is the time in years from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention,
- inIndicator contains the put/call and bought/sold information, and the up/down information for the **in barrier**, and
- outIndicator contains the put/call and bought/sold information, and the up/down information for the **out barrier**.

If the Valuation Date is greater than the **maturity date**, then the FX knock-in knock-out barrier option has expired and thus has a value of zero.

1.5.1 Representation Reduction

Equation (1.1) is only defined for the Strike representation. If the trade is specified by other representations, we need to reduce it to the Strike representation.

1.5.1.1 Default Representation

For the Default representation, the **strike rate**, X, is the ratio of CurrencyAmount and CrossCurrencyAmount, given by

$$X = \frac{N_{\rm p}}{N_{\rm c}}.\tag{1.2}$$

¹See FX Knock-In Knock-Out Barrier Option Pricing for details (p.12 of this document).

1.6 Examples

This section provides some deal examples of FX knock-in knock-out barrier option.

Example 1.1. An FX knock-in knock-out barrier option in Default representation:

- Currency: AUD
- CrossCurrency: GBP
- CurrencyAmount: 100,000,000
- CrossCurrencyAmount: 60,000,000
- InBarrier: 1.6305
- OutBarrier: 1.6725
- MaturityDate: 2013-11-15
- PutCall: Put
- InUpDown: Up
- OutUpDown: Up
- BoughtSold: Bought

Using equation (1.2), the **strike rate** of the option is

$$X = \frac{N_{\rm p}}{N_{\rm c}} = \frac{100,000,000}{60,000,000} = 1.6667$$

- a) If the AUD/GBP exchange rate never passed above the in barrier (1.6305) before 2013-11-15, the option was not knocked-in because the in barrier was not touched before the maturity date, thus the payoff of the option is 0.
- b) If on 2013-11-15, the option expires in the money with the AUD/GBP exchange rate being 1.6515, with the AUD/GBP exchange rate passing above the **in barrier** (1.6305) but not above the **out barrier** (1.6725) before 2013-11-15, the payoff of the option is

 $N_{\rm c} (X - S_T) = 60,000,000 \times (1.6667 - 1.6515) = $910,000$ AUD.

- c) If on 2013-11-15, the option expires out of the money with the AUD/GBP exchange rate being 1.6715, with the AUD/GBP exchange rate passing above the **in barrier** (1.6305) but not above the **out barrier** (1.6725) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (1.6667) is less than the AUD/GBP exchange rate (1.6715) on the **maturity date**.
- d) If the AUD/GBP exchange rate passed above the **out barrier** (1.6725) before 2013-11-15, the option was **knockedout** because the **out barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

Example 1.2. An FX knock-in knock-out barrier option in Default representation:

- Currency: AUD
- CrossCurrency: GBP
- CurrencyAmount: 100,000,000
- CrossCurrencyAmount: 60,000,000
- InBarrier: 1.6725
- OutBarrier: 1.6305
- MaturityDate: 2013-11-15
- PutCall: Put
- InUpDown: Up
- OutUpDown: Down
- BoughtSold: Bought

Using equation (1.2), the **strike rate** of the option is

$$X = \frac{N_{\rm p}}{N_{\rm c}} = \frac{100,000,000}{60,000,000} = 1.6667.$$

a) If the AUD/GBP exchange rate never went outside the range of the **out barrier** (1.6305) and the **in barrier** (1.6725) before 2013-11-15, the option was not **knocked-in** because the **in barrier** was not **touched** before the **maturity date**, thus the payoff of the option is 0.



- b) If the AUD/GBP exchange rate passed below the **out barrier** (1.6305) but not above the **in barrier** (1.6725) before 2013-11-15, the option was **knocked-out** because the **out barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.
- c) If on 2013-11-15, the option expires in the money with the AUD/GBP exchange rate being 1.6515, with the AUD/GBP exchange rate passing above the **in barrier** (1.6725) but not below the **out barrier** (1.6305) before 2013-11-15, the payoff of the option is

 $N_{\rm c} (X - S_T) = 60,000,000 \times (1.6667 - 1.6515) = \$910,000$ AUD.

- d) If on 2013-11-15, the option expires out of the money with the AUD/GBP exchange rate being 1.6715, with the AUD/GBP exchange rate passing above the **in barrier** (1.6725) but not below the **out barrier** (1.6305) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (1.6667) is less than the AUD/GBP exchange rate (1.6715) on the **maturity date**.
- e) If the AUD/GBP exchange rate passed below the **out barrier** (1.6305) and above the **in barrier** (1.6725) before 2013-11-15, the option was **knocked-out** because the **out barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

Example 1.3. An FX knock-in knock-out barrier option in Strike representation:

- Currency: JPY
- CrossCurrency: USD
- CrossCurrencyAmount: 100,000,000
- Strike: 100.2
- InBarrier: 97.5
- OutBarrier: 103.1
- MaturityDate: 2013-11-15
- PutCall: Call
- InUpDown: Down
- OutUpDown: Up
- BoughtSold: Bought
- a) If the USD/JPY exchange rate never went outside the range of the **in barrier** (97.5) and the **out barrier** (103.1) before 2013-11-15, the option was not **knocked-in** because the **in barrier** was not **touched** before the **maturity date**, thus the payoff of the option is 0.
- b) If the JPY/USD exchange rate passed above the **out barrier** (103.1) but not below the **in barrier** (97.5) before 2013-11-15, the option was **knocked-out** because the **out barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.
- c) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 102.5, and if the JPY/USD exchange rate passed below the **in barrier** (97.5) but not above the **out barrier** (103.1) before 2013-11-15, the payoff of the option is

$$N_{\rm c}(S_T - X) = 100,000,000 \times (102.5 - 100.2) = $230,000,000 \text{ JPY}.$$

- d) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing below the **in barrier** (97.5) but not above the **out barrier** (103.1) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.
- e) If the JPY/USD exchange rate passed above the **out barrier** (103.1) and below the **in barrier** (97.5) before 2013-11-15, the option was **knocked-out** because the **out barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

Example 1.4. An FX knock-in knock-out barrier option in Strike representation:

- Currency: JPY
- CrossCurrency: USD
- CrossCurrencyAmount: 100,000,000
- Strike: 100.2
- InBarrier: 103.1

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- OutBarrier: 97.5
- MaturityDate: 2013-11-15
- PutCall: Call
- InUpDown: Down
- OutUpDown: Down
- BoughtSold: Bought
- a) If the JPY/USD exchange rate never passed below the in barrier (103.1) before 2013-11-15, the option was not knocked-in because the in barrier was not touched before the maturity date, thus the payoff of the option is 0.
- b) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 103.5, and if the JPY/USD exchange rate passed below the **in barrier** (103.1) but not below the **out barrier** (97.5) before 2013-11-15, the payoff of the option is

 $N_{\rm c}(S_T - X) = 100,000,000 \times (103.5 - 100.2) = $330,000,000$ JPY.

- c) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing below the **in barrier** (103.1) but not below the **out barrier** (97.5) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.
- d) If the JPY/USD exchange rate passed below the **out barrier** (97.5) before 2013-11-15, the option was **knocked-out** because the **out barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

Chapter 2

FX Knock-In Knock-Out Barrier Option Pricing

2.1 Inputs to Function

Description	Symbol	min	max	$Reasonable\ range$
Spot rate of primary currency	$E_{\rm p}$	0^{+}	$+\infty$	
Spot rate of cross currency	$E_{\rm c}$	0^{+}	$+\infty$	
Strike rate as primary currency / cross currency	X	0^{+}	$+\infty$	
Cross currency amount	$N_{ m c}$	0^{+}	$+\infty$	
In barrier as primary currency/cross-currency	Ι	0^{+}	$+\infty$	
Out barrier as primary currency/cross-currency	U	0^{+}	$+\infty$	
Continuous zero rate of primary currency	$r_{ m p}$	0^{+}	$+\infty$	
Continuous zero rate of cross currency	$r_{ m c}$	0^{+}	$+\infty$	
Volatility of exchange rate between primary and cross currencies	σ	0^{+}	$+\infty$	
Time from value date to maturity in years	T	0^{+}	$+\infty$	
Put or Call		_	_	"P", "C"
Up or Down for in barrier	inIndicator	_	_	"U", "D"
Bought or Sold		_	_	"B", "S"
Up or Down for out barrier	outIndicator	_	_	"U", "D"

Table 2.1: Inputs for FX Knock-In Knock-Out Barrier Option pricing function

2.2 Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_{\rm c}}{E_{\rm p}}.$$

We can value an FX knock-in knock-out barrier option by calling the *knock-in knock-out barrier pricing function*¹ with appropriate inputs. The value of an FX knock-in knock-out barrier option in Base Currency is

$$N_{\rm c} \times E_{\rm p} \times \mathbb{I}_{\rm BS} \times {\rm KIKO}\left(S, X, I, O, r_{\rm p}, r_{\rm c}, \sigma, T, {\rm inIndicator}, {\rm outIndicator}\right),$$

where

$$\mathbb{I}_{\mathrm{BS}} = \begin{cases} 1, & \text{if indicator is `B',} \\ -1, & \text{if indicator is 'S'.} \end{cases}$$

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¹See pricing specification Knock-In Knock-Out Barrier Option for details.

Glossary

Base Currency The currency that the risk engine is configured to return values in.Reserve Currency The currency that all cross currency basis is benchmarked against.Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.