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# FX Single Barrier Option Product Specification

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## Chapter 1

# **FX Single Barrier Option**

### **1.1 Instrument Properties**

An FX single barrier option is a single barrier option with the **cross currency** as the underlying. If we denote the **primary currency** per **cross currency** exchange rate on the **maturity date** by  $S_T$ , and the agreed **strike rate** by X, provided the appropriate barrier condition is met:

1) for a knock-out type option, the barrier is not touched during the life of the option, or

2) for a **knock-in type** option, the **barrier** is **touched** during the life of the option,

and the option expires in the money, in which a call option expires in the money if  $S_T > X$  and a **put option** expires in the money if  $S_T < X$ , the holder of the call option (**put option**) buys (sells) the cross currency amount, denoted by  $N_c$ , at the predetermined strike rate X.

Provided either of the above barrier conditions holds, the payoff of an FX single barrier option is illustrated in Table 1.1.

Option Type	Expiry Condition	Payoff ( <b>primary currency</b> )
Call	$S_T \le X$ $S_T > X$	$\begin{matrix} 0\\ N_{\rm c} \left(S_T - X\right) \end{matrix}$
Put	$S_T \ge X$ $S_T < X$	$\begin{matrix} 0 \\ N_{\rm c} \left( X - S_T \right) \end{matrix}$

Table 1.1: Payoff at maturity for FX single barrier option if the underlying potion is active on the maturity date

## **1.2** Definitions

In this section, we define terms that are specific to FX single barrier options.

**barrier** is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** before or on the **maturity date**, the **underlying option** becomes active (inactive) for **knock-in type** (**knock-out type**) options.

barrier direction is the direction that the barrier is considered to be touched.

call option gives the holder the right, but not the obligation, to buy the cross currency at the strike rate on the maturity date if the option is knocked-in (not knocked-out) for a knock-in type (knock-out type) option.

**cross currency** is the currency nominated as the underlying asset.

cross currency amount is the deal amount in cross currency that will be exchanged if the option is exercised.



- down is the barrier direction in cases where if the primary currency per cross currency exchange rate passes below the barrier before or on the maturity date, the barrier is considered to be touched.
- **knocked-in** applies to **knock-in type** options and means the **barrier** was **touched** and the **underlying option** became active.
- **knocked-out** applies to **knock-out type** options and means the **barrier** was **touched** and the **underlying option** became inactive.
- knock-in type means the underlying option only becomes active if the barrier is touched before or on the maturity date.
- knock-out type means the underlying option becomes inactive if the barrier is touched before or on the maturity date.
- maturity date is the date the option expires.
- **primary currency** is the currency that the deal is quoted in.
- put option gives the holder the right, but not the obligation, to sell the **cross currency** at the **strike rate** on the **maturity date** if the option is **knocked-in** (not **knocked-out**) for a **knock-in type** (**knock-out type**) option.
- settlement date is the date the cross currency amount is exchanged at the strike rate if the option is exercised.
- strike rate is the agreed exchange rate between primary currency and cross currency if the option is exercised, quoted in primary currency per cross currency.
- touched means for an option with an up (down) barrier, the primary currency per cross currency exchange rate was above (below) the barrier before or on the maturity date.
- **underlying option** is the underlying FX vanilla option that specifies the payoff of the option should the option be either **knocked-in** or not **knocked-out**.
- up is the barrier direction in cases where if the primary currency per cross currency exchange rate passes above the barrier before or on the maturity date, the barrier is considered to be touched.

### **1.3** Representations

In the Risk Engine, products are specified by *representations*. In this section, we provide the representations of FX single barrier options.

#### **1.3.1** Default Representation

The *Default* representation consists of the mandatory trade fields in Table 1.2, the optional trade field in Table 1.3, with their restrictions in Table 1.4.

#### 1.3.1.1 Required Curves

The following curves are required by an FX single barrier option:

- Currency FX spot curve: FX Spot Curve (FX.PRICE.Currency.BaseCurrency),
- CrossCurrency FX spot curve: FX Spot Curve (FX.PRICE.CrossCurrency.BaseCurrency),
- Currency discounting curve: FX Zero Curve (FX.ZERO.Currency.ReserveCurrency),
- CrossCurrency discounting curve: FX Zero Curve (FX.ZERO.CrossCurrency.ReserveCurrency), and
- Currency, CrossCurrency volatility grid: FX Volatility Grid (FX.GRID.CrossCurrency.Currency).

#### 1.3.2 Strike Representation

The *Strike* representation consists of the mandatory trade fields in Table 1.5, the optional trade field in Table 1.6, with their restrictions in Table 1.7.



Field	Description	Data Type	Symbol
Currency	The primary currency	string	р
CrossCurrency	The cross currency	string	с
CurrencyAmount	The deal amount in <i>Currency</i>	double	$N_{\rm p}$
CrossCurrencyAmount	The deal amount in CrossCurrency, i.e., the cross	double	$\dot{N_{c}}$
	currency amount		
Barrier	The <b>barrier</b> level as <i>Currency/CrossCurrency</i>	double	H
MaturityDate	The maturity date	date	MD
PutCall	Put option or call option on CrossCurrency	string	$\mathbf{PC}$
UpDown	Direction of the barrier	string	UD
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.2: Mandatory trade fields for the Default representation of the FX Single Barrier Option

Field	Description	Data Type	Symbol	Default Value
SettlementDate	The settlement date $\mathbf{A}$	date	SD	MD

Table 1.3: Optional trade field for the Default representation of the FX Single Barrier Option

Field	Restriction
CrossCurrency	$c \neq p$
CurrencyAmount	$N_{\rm p} > 0$
CrossCurrencyAmount	$N_{\rm c} > 0$
Barrier	H > 0
PutCall	Put, Call, P, C
UpDown	Up, Down, U, D
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S
SettlementDate	$SD \ge MD$

Table 1.4: Trade field restrictions for the Default representation of the FX Single Barrier Option

Field	Description	Data Type	Symbol
Currency	The primary currency	string	р
CrossCurrency	The cross currency	string	с
CrossCurrencyAmount	The deal amount in CrossCurrency, i.e., the cross	double	$N_{\rm c}$
	currency amount		
Strike	The <b>strike rate</b> as <i>Currency</i> / <i>CrossCurrency</i>	double	X
Barrier	The <b>barrier</b> level as <i>Currency/CrossCurrency</i>	double	H
MaturityDate	The maturity date	date	MD
PutCall	Put option or call option on CrossCurrency	string	$\mathbf{PC}$
UpDown	Direction of the barrier	string	UD
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.5: Mandatory trade fields for the Strike representation of the FX Single Barrier Option

#### 1.3.2.1 Required Curves

The following curves are required by an FX single barrier option:



Field	Description	Data Type	Symbol	Default Value
SettlementDate	The settlement date	date	SD	MD

Table 1.6: Optional trade field for the Strike representation of the FX Single Barrier Option

Field	Restriction
CrossCurrency	$\mathbf{c} \neq \mathbf{p}$
CrossCurrencyAmount	$N_{\rm c} > 0$
Strike	X > 0
Barrier	H > 0
PutCall	Put, Call, P, C
UpDown	Up, Down, U, D
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S
SettlementDate	$SD \ge MD$

Table 1.7: Trade field restrictions for the Strike representation of the FX Single Barrier Option

- Currency FX spot curve: FX Spot Curve (FX.PRICE.Currency.BaseCurrency),
- CrossCurrency FX spot curve: FX Spot Curve (FX.PRICE.CrossCurrency.BaseCurrency),
- Currency discounting curve: FX Zero Curve (FX.ZERO.Currency.ReserveCurrency),
- CrossCurrency discounting curve: FX Zero Curve (FX.ZERO.CrossCurrency.ReserveCurrency), and
- Currency, CrossCurrency volatility grid: FX Volatility Grid (FX.GRID.CrossCurrency.Currency).

### 1.4 Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX single barrier option in Base Currency is given by the FX single barrier option pricing function<sup>1</sup>,

$$FXSingleBarrier(E_{p}, E_{c}, X, N_{c}, H, r_{p}, r_{c}, \sigma, T, indicator),$$
(1.1)

where

- $E_{\rm p}$  is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,
- $E_{\rm c}$  is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,
- X is the strike rate in units of primary currency per cross currency,
- $N_{\rm c}$  is the cross currency amount,
- *H* is the **barrier** level in units of **primary currency** per **cross currency**,
- r<sub>p</sub> is the cross currency basis adjusted continuous zero rate of primary currency from Valuation Date to maturity date in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{\rm c}$  is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- $\sigma$  is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,



<sup>&</sup>lt;sup>1</sup>See FX Single Barrier Option Pricing for details (p.10 of this document).

- T is the time in years from Valuation Date to maturity date in Actual/365 (Fixed) day count convention, and
- indicator contains the put/call, up/down, in/out and bought/sold information.

If the Valuation Date is greater than the **maturity date**, then the FX single barrier option has expired and thus has a value of zero.

#### 1.4.1 Representation Reduction

Equation (1.1) is only defined for the Strike representation. If the trade is specified by other representations, we need to reduce it to the Strike representation.

#### 1.4.1.1 Default Representation

For the Default representation, the **strike rate**, X, is the ratio of CurrencyAmount and CrossCurrencyAmount, given by

$$X = \frac{N_{\rm p}}{N_{\rm c}}.\tag{1.2}$$

### 1.5 Examples

This section provides some deal examples of FX single barrier option.

Example 1.1. An FX single barrier option in Default representation:

- Currency: AUD
- CrossCurrency: GBP
- CurrencyAmount: 100,000,000
- CrossCurrencyAmount: 60,000,000
- Barrier: 1.6725
- MaturityDate: 2013-11-15
- PutCall: Put
- UpDown: Up
- InOut: Out
- BoughtSold: Bought

Using equation (1.2), the **strike rate** of the option is

$$X = \frac{N_{\rm p}}{N_{\rm c}} = \frac{100,000,000}{60,000,000} = 1.6667$$

a) If on 2013-11-15, the option expires money with the AUD/GBP exchange rate being 1.6515, with the AUD/GBP exchange rate never passing above the **barrier** (1.6725) before 2013-11-15, the payoff of the option is

 $N_{\rm c} (X - S_T) = 60,000,000 \times (1.6667 - 1.6515) =$ \$910,000 AUD.

- b) If on 2013-11-15, the option expires out of the money with the AUD/GBP exchange rate being 1.6715, with the AUD/GBP exchange rate never passing above the **barrier** (1.6725) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (1.6667) is less than the AUD/GBP exchange rate (1.6715) on the **maturity date**.
- c) If the AUD/GBP exchange rate passed above the **barrier** (1.6725) before 2013-11-15, the option was **knocked-out** because the **barrier** was **touched** before the **maturity date**, thus the payoff of the option is 0.

**Example 1.2.** An FX single barrier option in Strike representation:

- Currency: JPY
- CrossCurrency: USD
- CrossCurrencyAmount: 100,000,000
- Strike: 100.2
- Barrier: 97.5
- MaturityDate: 2013-11-15
- PutCall: Call
- UpDown: Down
- InOut: In

- BoughtSold: Bought
- a) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 102.5, with the JPY/USD exchange rate passing below the **barrier** (97.5) before 2013-11-15, the payoff of the option is

 $N_{\rm c} (S_T - X) = 100,000,000 \times (102.5 - 100.2) = $230,000,000$  USD.

- b) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing below the **barrier** (97.5) before 2013-11-15, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.
- c) If the JPY/USD exchange rate never passed below the **barrier** (97.5) before 2013-11-15, the option was not **knocked-in** because the **barrier** was not **touched** before the **maturity date**, thus the payoff of the option is 0.

# Chapter 2

# **FX Single Barrier Option Pricing**

## 2.1 Inputs to Function

Description	Symbol	min	max	Reasonable range
Spot rate of primary currency	$E_{\rm p}$	$0^{+}$	$+\infty$	
Spot rate of cross currency	$\dot{E_{c}}$	$0^{+}$	$+\infty$	
Strike rate as primary currency / cross currency	X	$0^{+}$	$+\infty$	
Cross currency amount	$N_{\rm c}$	$0^{+}$	$+\infty$	
Barrier as primary currency/cross-currency	H	$0^{+}$	$+\infty$	
Continuous zero rate of primary currency	$r_{ m p}$	$0^{+}$	$+\infty$	
Continuous zero rate of cross currency	$r_{c}$	$0^{+}$	$+\infty$	
Volatility of exchange rate between primary and cross currencies	$\sigma$	$0^{+}$	$+\infty$	
Time from value date to maturity in years	T	$0^{+}$	$+\infty$	
Put or Call		_	_	"P", "C"
Up or Down	• 1• /	_	_	"U", "D"
In or Out	indicator	_	_	"I", "O"
Bought or Sold		_	_	"B", "S"

Table 2.1: Inputs for FX Single Barrier Option pricing function

## 2.2 Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_{\rm c}}{E_{\rm p}}.$$

We can value an FX single barrier option by calling the *single barrier pricing function*<sup>1</sup> with appropriate inputs. The value of an FX single barrier option in Base Currency is

 $N_{\rm c} \times E_{\rm p} \times \mathbb{I}_{\rm BS} \times {\rm SingleBarrier}\left(S, X, H, 0, r_{\rm p}, r_{\rm c}, \sigma, T, {\rm indicator}\right),$ 

where

$$\mathbb{I}_{BS} = \begin{cases} 1, & \text{if indicator is 'B',} \\ -1, & \text{if indicator is 'S'.} \end{cases}$$

<sup>&</sup>lt;sup>1</sup>See pricing specification *Single Barrier Option* for details.

# Glossary

Base Currency The currency that the risk engine is configured to return values in.Reserve Currency The currency that all cross currency basis is benchmarked against.Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.