FX Single Window Barrier Option Product Specification

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Chapter 1

FX Single Window Barrier Option

1.1 Instrument Properties

An FX single window barrier option is a single window barrier option with the **cross currency** as the underlying. If we denote the **primary currency** per **cross currency** exchange rate on the **maturity date** by S_T, and the agreed **strike rate** by X, provided the appropriate barrier condition is met:

1) for a **knock-out type** option, the **barrier** is not **touched** between the **barrier start date** and the **barrier end date**, or

2) for a **knock-in type** option, the **barrier** is **touched** between the **barrier start date** and the **barrier end date**,

and the option expires in the money, in which a **call option** expires in the money if $S_T > X$ and a **put option** expires in the money if $S_T < X$, the holder of the **call option** (**put option**) buys (sells) the **cross currency amount**, denoted by N_c, at the predetermined **strike rate** X.

Provided either of the above barrier conditions holds, the payoff of an FX single window barrier option is illustrated in Table 1.1.

Option Type	Expiry Condition	Payoff (**primary currency**)
Call	$S_T \leq X$	0
	$S_T > X$	$N_c\,(S_T - X)$
Put	$S_T \geq X$	0
	$S_T < X$	$N_c\,(X - S_T)$

Table 1.1: Payoff at maturity for FX single window barrier option if the underlying potion is active on the maturity date

1.2 Definitions

In this section, we define terms that are specific to FX single window barrier options.

barrier is the **primary currency** per **cross currency** exchange rate level such that, if it is **touched** between the **barrier start date** and the **barrier end date**, the **underlying option** becomes active (inactive) for **knock-in type** (**knock-out type**) options.

barrier direction is the direction that the **barrier** is considered to be **touched**.

barrier end date is the date the **barrier** becomes inactive.

barrier start date is the date the **barrier** becomes active.

call option gives the holder the right, but not the obligation, to buy the **cross currency** at the **strike rate** on the **maturity date** if the option is **knocked-in** (not **knocked-out**) for a **knock-in type** (**knock-out type**) option.

cross currency is the currency nominated as the underlying asset.

cross currency amount is the deal amount in **cross currency** that will be exchanged if the option is exercised.

down is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes below the **barrier** between the **barrier start date** and the **barrier end date**, the **barrier** is considered to be **touched**.

knocked-in applies to **knock-in type** options and means the **barrier** was **touched** and the **underlying option** became active.

knocked-out applies to **knock-out type** options and means the **barrier** was **touched** and the **underlying option** became inactive.

knock-in type means the **underlying option** only becomes active if the **barrier** is **touched** between the **barrier start date** and the **barrier end date**.

knock-out type means the **underlying option** becomes inactive if the **barrier** is **touched** between the **barrier start date** and the **barrier end date**.

maturity date is the date the option expires.

primary currency is the currency that the deal is quoted in.

put option gives the holder the right, but not the obligation, to sell the **cross currency** at the **strike rate** on the **maturity date** if the option is **knocked-in** (not **knocked-out**) for a **knock-in type** (**knock-out type**) option.

strike rate is the agreed exchange rate between **primary currency** and **cross currency** if the option is exercised, quoted in **primary currency** per **cross currency**.

touched means for an option with an **up** (**down**) barrier, the **primary currency** per **cross currency** exchange rate was above (below) the **barrier** between the **barrier start date** and the **barrier end date**.

underlying option is the underlying FX vanilla option that specifies the payoff of the option should the option be either **knocked-in** or not **knocked-out**.

up is the **barrier direction** in cases where if the **primary currency** per **cross currency** exchange rate passes above the **barrier** between the **barrier start date** and the **barrier end date**, the **barrier** is considered to be **touched**.

1.3 Representations

In the Risk Engine, products are specified by *representations*. In this section, we provide the representations of FX single window barrier options.

1.3.1 Default Representation

The *Default* representation consists of the mandatory trade fields in Table 1.2, with their restrictions in Table 1.3.

1.3.1.1 Required Curves

The following curves are required by an FX single window barrier option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),
- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),
- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),
- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and
- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CurrencyAmount	The deal amount in *Currency*	double	N_{p}
CrossCurrencyAmount	The deal amount in *CrossCurrency*, *i.e.*, the **cross currency amount**	double	N_{c}
Barrier	The **barrier** level as *Currency*/*CrossCurrency*	double	H
BarrierStart	The **barrier start date**	date	BSD
BarrierEnd	The **barrier end date**	date	BED
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
UpDown	Direction of the barrier	string	UD
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.2: Mandatory trade fields for the Default representation of the FX Single Window Barrier Option

Field	*Restriction*
CrossCurrency	$\mathrm{c} \neq \mathrm{p}$
CurrencyAmount	$N_{\mathrm{p}} > 0$
CrossCurrencyAmount	$N_{\mathrm{c}} > 0$
Barrier	$H > 0$
BarrierEnd	BED > BSD
MaturityDate	MD > BED
PutCall	Put, Call, P, C
UpDown	Up, Down, U, D
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S

Table 1.3: Trade field restrictions for the Default representation of the FX Single Window Barrier Option

Field	*Description*	*Data Type*	*Symbol*
Currency	The **primary currency**	string	p
CrossCurrency	The **cross currency**	string	c
CrossCurrencyAmount	The deal amount in *CrossCurrency*, *i.e.*, the **cross currency amount**	double	N_{c}
Strike	The **strike rate** as *Currency*/*CrossCurrency*	double	X
Barrier	The **barrier** level as *Currency*/*CrossCurrency*	double	H
BarrierStart	The **barrier start date**	date	BSD
BarrierEnd	The **barrier end date**	date	BED
MaturityDate	The **maturity date**	date	MD
PutCall	Put option or call option on *CrossCurrency*	string	PC
UpDown	Direction of the barrier	string	UD
InOut	Knock-in option or knock-out option	string	IO
BoughtSold	Bought or sold the option	string	BS

Table 1.4: Mandatory trade fields for the Strike representation of the FX Single Window Barrier Option

1.3.2 Strike Representation

The *Strike* representation consists of the mandatory trade fields in Table 1.4, with their restrictions in Table 1.5.

Field	*Restriction*
CrossCurrency	$c \neq p$
CrossCurrencyAmount	$N_c > 0$
Strike	$X > 0$
Barrier	$H > 0$
BarrierEnd	BED > BSD
MaturityDate	MD > BED
PutCall	Put, Call, P, C
UpDown	Up, Down, U, D
InOut	In, Out, I, O
BoughtSold	Bought, Sold, B, S

Table 1.5: Trade field restrictions for the Strike representation of the FX Single Window Barrier Option

1.3.2.1 Required Curves

The following curves are required by an FX single window barrier option:

- *Currency FX spot curve*: FX Spot Curve — (FX.PRICE.Currency.BaseCurrency),
- *CrossCurrency FX spot curve*: FX Spot Curve — (FX.PRICE.CrossCurrency.BaseCurrency),
- *Currency discounting curve*: FX Zero Curve — (FX.ZERO.Currency.ReserveCurrency),
- *CrossCurrency discounting curve*: FX Zero Curve — (FX.ZERO.CrossCurrency.ReserveCurrency), and
- *Currency, CrossCurrency volatility grid*: FX Volatility Grid — (FX.GRID.CrossCurrency.Currency).

1.4 Formula

If the Valuation Date is less than or equal to the **maturity date**, the value of an FX single window barrier option in Base Currency is given by the *FX single window barrier option pricing function*[1],

$$\text{FXSingleWindowBarrier}\left(E_p, E_c, X, N_c, H, r_{p,1}, r_{c,1}, \sigma_1, t_1, r_{p,2}, r_{c,2}, \sigma_2, t_2, r_{p,3}, r_{c,3}, \sigma_3, T_3, \text{indicator}\right), \tag{1.1}$$

where

- E_p is the spot exchange rate in units of Base Currency per **primary currency**, from the Currency FX spot curve,
- E_c is the spot exchange rate in units of Base Currency per **cross currency**, from the CrossCurrency FX spot curve,
- X is the **strike rate** in units of **primary currency** per **cross currency**,
- N_c is the **cross currency amount**,
- H is the **barrier** in units of **primary currency** per **cross currency**,
- $r_{p,1}$ is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{c,1}$ is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ_1 is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- t_1 is the time in years from Valuation Date to **barrier start date** in Actual/365 (Fixed) day count convention,

[1] See FX Single Window Barrier Option Pricing for details (p.10 of this document).

- $r_{p,2}$ is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{c,2}$ is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ_2 is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- t_2 is the time in years from Valuation Date to **barrier end date** in Actual/365 (Fixed) day count convention,
- $r_{p,3}$ is the cross currency basis adjusted continuous zero rate of **primary currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency discounting curve,
- $r_{c,3}$ is the cross currency basis adjusted continuous zero rate of **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the CrossCurrency discounting curve,
- σ_3 is the volatility of the exchange rate between **primary currency** and **cross currency** from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, from the Currency, CrossCurrency volatility grid,
- T_3 is the time in years from Valuation Date to **maturity date** in Actual/365 (Fixed) day count convention, and
- indicator contains the put/call, up/down, in/out and bought/sold information.

If the Valuation Date is greater than the **maturity date**, then the FX single window barrier option has expired and thus has a value of zero.

1.4.1 Representation Reduction

Equation (1.1) is only defined for the Strike representation. If the trade is specified by other representations, we need to reduce it to the Strike representation.

1.4.1.1 Default Representation

For the Default representation, the **strike rate**, X, is the ratio of CurrencyAmount and CrossCurrencyAmount, given by

$$X = \frac{N_p}{N_c}. \tag{1.2}$$

1.5 Examples

This section provides some deal examples of FX single window barrier option.

Example 1.1. An FX single window barrier option in Default representation:

- Currency: AUD
- CrossCurrency: GBP
- CurrencyAmount: 100,000,000
- CrossCurrencyAmount: 60,000,000
- Barrier: 1.6725
- BarrierStartDate: 2013-10-15
- BarrierEndDate: 2013-10-31
- MaturityDate: 2013-11-15
- PutCall: Put
- UpDown: Up
- InOut: Out
- BoughtSold: Bought

Using equation (1.2), the **strike rate** of the option is

$$X = \frac{N_{\mathrm{p}}}{N_{\mathrm{c}}} = \frac{100,000,000}{60,000,000} = 1.6667.$$

a) If on 2013-11-15, the option expires in the money with the AUD/GBP exchange rate being 1.6515, with the AUD/GBP exchange rate never passing above the **barrier** (1.6725) between 2013-10-15 and 2013-10-31, the payoff of the option is

$$N_{\mathrm{c}}\left(X - S_T\right) = 60,000,000 \times (1.6667 - 1.6515) = \$910,000 \text{ AUD}.$$

b) If on 2013-11-15, the option expires out of the money with the AUD/GBP exchange rate being 1.6715, with the AUD/GBP exchange rate never passing above the **barrier** (1.6725) between 2013-10-15 and 2013-10-31, the payoff of the option is 0 as the **strike rate** (1.6667) is less than the AUD/GBP exchange rate (1.6715) on the **maturity date**.

c) If the AUD/GBP exchange rate passed above the **barrier** (1.6725) between 2013-10-15 and 2013-10-31, the option was **knocked-out** because the **barrier** was **touched** between the **barrier start date** and the **barrier end date**, thus the payoff of the option is 0.

Example 1.2. An FX single window barrier option in Strike representation:

- Currency: JPY
- CrossCurrency: USD
- CrossCurrencyAmount: 100,000,000
- Strike: 100.2
- Barrier: 97.5
- BarrierStartDate: 2013-10-15
- BarrierEndDate: 2013-10-31
- MaturityDate: 2013-11-15
- PutCall: Call
- UpDown: Down
- InOut: In
- BoughtSold: Bought

a) If on 2013-11-15, the option expires in the money with the JPY/USD exchange rate being 102.5, with the JPY/USD exchange rate passing below the **barrier** (97.5) between 2013-10-15 and 2013-10-31, the payoff of the option is

$$N_{\mathrm{c}}\left(S_T - X\right) = 100,000,000 \times (102.5 - 100.2) = \$230,000,000 \text{ JPY}.$$

b) If on 2013-11-15, the option expires out of the money with the JPY/USD exchange rate being 98.4, with the JPY/USD exchange rate passing below the **barrier** (97.5) between 2013-10-15 and 2013-10-31, the payoff of the option is 0 as the **strike rate** (100.2) is greater than the JPY/USD exchange rate (98.4) on the **maturity date**.

c) If the JPY/USD exchange rate never passed below **barrier** (97.5) between 2013-10-15 and 2013-10-31, the option was not **knocked-in** because the **barrier** was not **touched** between the **barrier start date** and the **barrier end date**, thus the payoff of the option is 0.

Chapter 2

FX Single Window Barrier Option Pricing

2.1 Inputs to Function

Description	*Symbol*	*min*	*max*	*Reasonable range*
Spot rate of primary currency	E_{p}	0^+	$+\infty$	
Spot rate of cross currency	E_{c}	0^+	$+\infty$	
Strike rate as primary currency/cross currency	X	0^+	$+\infty$	
Cross currency amount	N_{c}	0^+	$+\infty$	
Barrier as primary currency/cross-currency	H	0^+	$+\infty$	
Continuous zero rate of primary currency till t_1	$r_{\mathrm{p},1}$	0^+	$+\infty$	
Continuous zero rate of cross currency till t_1	$r_{\mathrm{c},1}$	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies till t_1	σ_1	0^+	$+\infty$	
Time from value date to barrier start date in years	t_1	0^+	$< t_2$	
Continuous zero rate of primary currency till t_2	$r_{\mathrm{p},2}$	0^+	$+\infty$	
Continuous zero rate of cross currency till t_2	$r_{\mathrm{c},2}$	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies till t_2	σ_2	0^+	$+\infty$	
Time from value date to barrier end date in years	t_2	$> t_1$	$< T_3$	
Continuous zero rate of primary currency till T_3	$r_{\mathrm{p},3}$	0^+	$+\infty$	
Continuous zero rate of cross currency till T_3	$r_{\mathrm{c},3}$	0^+	$+\infty$	
Volatility of exchange rate between primary and cross currencies till T_3	σ_3	0^+	$+\infty$	
Time from value date to maturity in years	T_3	$> t_2$	$+\infty$	
Put or Call	indicator	–	–	"P", "C"
Up or Down		–	–	"U", "D"
In or Out		–	–	"I", "O"
Bought or Sold		–	–	"B", "S"

Table 2.1: Inputs for FX Single Window Barrier Option pricing function

2.2 Formula

The spot exchange rate of primary currency per cross currency is given by

$$S = \frac{E_{\mathrm{c}}}{E_{\mathrm{p}}}.$$

We can value an FX single window barrier option by calling the *single window barrier pricing function*[1] with appropriate inputs. The value of an FX single window barrier option in Base Currency is

$$N_{\mathrm{c}} \times E_{\mathrm{p}} \times \mathbb{I}_{\mathrm{BS}} \times \mathrm{SingleWindowBarrier}\left(S, X, H, r_{\mathrm{p},1}, r_{\mathrm{c},1}, \sigma_1, t_1, r_{\mathrm{p},2}, r_{\mathrm{c},2}, \sigma_2, t_2, r_{\mathrm{p},3}, r_{\mathrm{c},3}, \sigma_3, T_3, \mathrm{indicator}\right),$$

where

$$\mathbb{I}_{\mathrm{BS}} = \begin{cases} 1, & \text{if indicator is 'B',} \\ -1, & \text{if indicator is 'S'.} \end{cases}$$

[1] See pricing specification *Single Window Barrier Option* for details.

Glossary

Base Currency The currency that the risk engine is configured to return values in.

Reserve Currency The currency that all cross currency basis is benchmarked against.

Risk Engine The Vector Risk market risk and credit risk system.

Valuation Date The date that we value the trades as.